KERALA CEE

Engineering Entrance Exam

Solved Paper 2022

Physics

1. The dimensional formula for the power of a lens is

(a) $[L^{-1}M^0T^0]$ (b) $[L^0M^{-1}T^0]$
(c) $[L^0M^0T^{-1}]$ (d) $[L^0M^0T^0]$
(e) $[L^{-1}M^0T^{-1}]$

2. The technology related with the Bernoulli's principle is used in

(a) hydroelectric power (b) rocket propulsion
(c) aeroplane (d) steam engine
(e) electron microscope

3. The final result of the sum of the numbers 523.32, 1.21524 and 107.3 rounded to correct significant figures is

(a) 631.8 (b) 631.835
(c) 631.83 (d) 631.8352
(e) 631.83524

4. A cyclist starting from rest moves with uniform acceleration and covers 120 m in 10 s, then his acceleration (in ms^{-2}) is

(a) 5 (b) 1.5 (c) 2.4 (d) 3
(e) 4.8

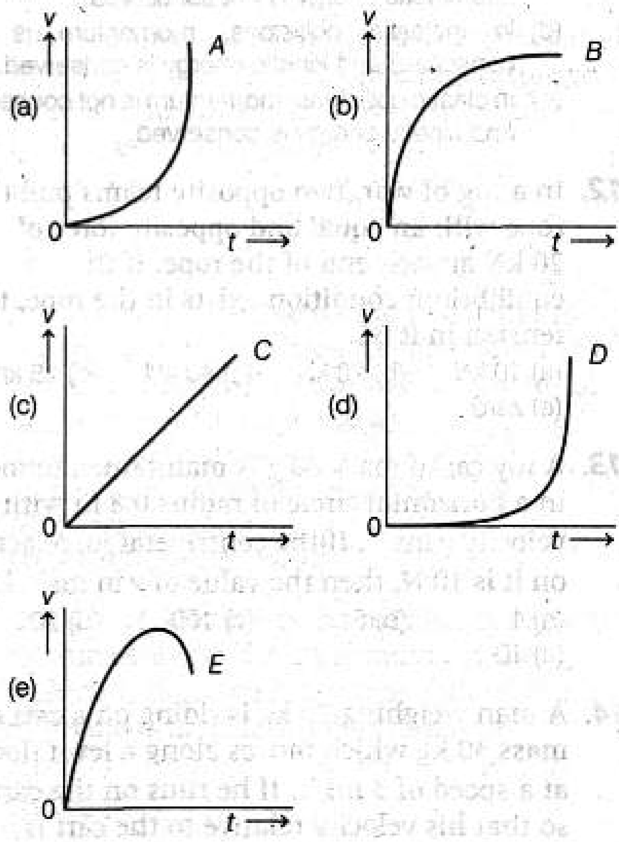
5. The angle made by $\mathbf{r} = 3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ with the X-axis is

(a) 30° (b) 60° (c) 180° (d) 90°
(e) 45°

6. In projectile motion, the physical quantity that remains invariant throughout is

(a) vertical component of velocity
(b) horizontal component of velocity
(c) kinetic energy of the projectile
(d) potential energy of the projectile
(e) linear momentum of the projectile

7. Given below are the velocity-time graphs of five particles A, B, C, D and E. The correct graph from the following v-t plots in which the velocity of the particle is a function of t^2 is



8. Which one of the following is not a contact force?

(a) Frictional force (b) Buoyant force
(c) Air resistance (d) Viscous force
(e) Gravitational force

9. Two bodies of masses m and $4m$ have kinetic energies in the ratio 1 : 2. Their momenta p_1 and p_2 are in the ratio

(a) $1:2\sqrt{2}$
(b) $1:2\sqrt{3}$
(c) $2\sqrt{2}:1$
(d) $3\sqrt{2}:1$
(e) $1:3\sqrt{2}$

10. Work-energy theorem is an integral form of

(a) Newton's first law
(b) law of equipartition of energy
(c) Newton's second law
(d) Newton's law of gravitation
(e) Newton's third law

11. Which one of the following statement is correct?

(a) In inelastic collisions, both momentum and kinetic energy are conserved.
(b) In inelastic collisions, momentum is conserved and kinetic energy is not conserved.
(c) In elastic collisions, momentum is conserved and kinetic energy is not conserved.
(d) In inelastic collisions, momentum is not conserved and kinetic energy is conserved.
(e) In elastic collisions, momentum is not conserved and kinetic energy is conserved.

12. In a tug of war, two opposite teams pull the rope with an equal and opposite force of 20 kN at each end of the rope. If the equilibrium condition exists in the rope, the tension in it is

(a) 10 kN (b) 20 kN (c) 40 kN (d) 15 kN
(e) zero

13. A toy car of mass 80 g is maintained to move in a horizontal circle of radius 0.8 m with a velocity v ms^{-1}. If the centripetal force acting on it is 10 N, then the value of v in ms^{-1} is

(a) 1 (b) 5 (c) 100 (d) 20
(e) 10

14. A man weighing 70 kg is riding on a cart of mass 30 kg which moves along a level floor at a speed of 3 ms^{-1}. If he runs on the cart, so that his velocity relative to the cart is 4 ms^{-1} in the direction opposite to the motion of the cart, the speed of centre of mass of the system is

(a) 0.3 ms^{-1} (b) 0.5 ms^{-1} (c) 0.2 ms^{-1} (d) 0.1 ms^{-1}
(e) zero

15. Two persons stand at the edges of a rotating circular platform at diametrically opposite points. If they start moving towards each other at uniform velocity, then its

(a) angular velocity decreases
(b) moment of inertia increases
(c) moment of inertia remains constant
(d) angular velocity increases and moment of inertia decreases
(e) both angular velocity and moment of inertia remain constant

16. A solid metal cylinder A and a hollow metal cylinder B have same mass but their radii are in the ratio 2 : 1. Then, the ratio of their respective moments of inertia about their own axis is

(a) 1 : 1
(b) 2 : 1
(c) 4 : 1
(d) 1 : 4
(e) 1 : 2

17. The angular momentum of a particle with respect to the origin will not be zero, if

(a) the directional line of linear momentum passes through the origin
(b) the particle is at the origin
(c) the angle between the position vector and linear momentum is 180°
(d) the linear momentum vanishes
(e) the angle between the position vector and linear momentum is 90°

18. The minimum speed at which an object of 1 kg mass is thrown from the surface of the moon, so that it does not fall back to the moon is

(a) 2.3 km/h
(b) 3.2 km/h
(c) 11.2 km/h
(d) 1.2 km/s
(e) 2.3 km/s

19. Weight of a body of mass m in its free fall above the surface of the earth is

(a) mg (b) $\sqrt{mg}$ (c) infinity (d) $m\sqrt{g}$
(e) zero

20. Two satellites A and B are orbiting a planet in circular orbits with radii $2R$ and R, respectively. If the speed of satellite A is $2v$, then the speed of satellite B is

(a) $6\sqrt{2}\ v$ (b) $2\sqrt{2}\ v$ (c) $5\sqrt{2}\ v$ (d) $6v$
(e) $4v$

21. Gravitational potential energy associated with two point masses, each of 1 kg, separated by a distance of 1 cm (in joule) is (G = gravitational constant)

(a) 2G (b) 100G
(c) 1000G (d) G
(e) 500G

22. The relative viscosity of blood remains constant between

(a) 0°C and 37°C (b) 30°C and 59°C
(c) 10°C and 47°C (d) 0°C and 57°C
(e) 20°C and 47°C

23. If the Young's modulus of the material of a wire is numerically equal to ten times the stress applied to a wire of length l, then the change in the length of the wire is

(a) $0.1\,l$ (b) $0.5\,l$ (c) $0.2\,l$ (d) $0.75\,l$
(e) $0.25\,l$

24. The working of hydraulic lift is based on the principle of

(a) Bernoulli principle (b) Toricelli's law
(c) Pascal's law (d) Magnus effect
(e) Stoke's law

25. An ideal Carnot engine working with source temperature T_1 and sink temperature T_2, has efficiency η. Then the value of the ratio $\frac{T_1}{T_2}$ is

(a) $\frac{1}{1-\eta}$ (b) $\frac{1-\eta}{1}$ (c) $\frac{1}{\eta}$ (d) η
(e) $\frac{\eta}{1-\eta}$

26. A process in which the amount of heat supplied to the system goes fully to change its internal energy and temperature is

(a) adiabatic process (b) cyclic process
(c) isobaric process (d) isothermal process
(e) isochoric process

27. The incorrect statement is

(a) a liquid is incompressible and has free surface of its own.
(b) a gas is compressible and occupy all the space available to it.
(c) pressure in a fluid at rest is same at all points which are at the same height.
(d) the surface of water in a capillary is concave.
(e) surface tension is a force per unit area.

28. Three identical silver cups A, B and C contain three liquids of same densities at same temperature higher than the temperature of the surrounding. If the ratio of their specific heat capacities is 1 : 2 : 4, then

(a) A cools faster than B but slower than C
(b) B cools faster than C but slower than A
(c) A cools faster than B and C
(d) C cools faster than B and A
(e) B cools faster than A and C

29. A polyatomic molecule has 3 translational, 3 rotational degrees of freedom and 2 vibrational modes. The ratio of specific heats $\frac{C_p}{C_V}$ is

(a) $\frac{7}{5}$ (b) $\frac{3}{5}$ (c) $\frac{5}{6}$ (d) $\frac{5}{3}$
(e) $\frac{6}{5}$

30. The rms speed of molecules of an ideal gas at 27°C is 200 ms^{-1}. When the temperature is increased to 327°C, the rms speed of the molecules is changed to

(a) 490.2 ms^{-1} (b) 315.2 ms^{-1}
(c) 282.8 ms^{-1} (d) 425.5 ms^{-1}
(e) 515.7 ms^{-1}

31. No process is possible whose sole result is the transfer of heat from a colder object to a hotter object. This is Clausius statement for

(a) zeroth law of thermodynamics
(b) first law of thermodynamics
(c) second law of thermodynamics
(d) carnot's theorem
(e) principle of refrigeration

32. In a gas at STP, if n is the number density of the molecules and r is the radius of the molecule, then the mean free path of the molecule is inversely proportional to

(a) nr^2 (b) nr (c) n^2r (d) $\sqrt{nr}$
(e) $\sqrt{nr}$

33. A tuning fork produces 4 beats per second with both 26.0 cm and 25.2 cm of stretched sonometer wire. Frequency of the fork is

(a) 285 Hz (b) 384 Hz
(c) 512 Hz (d) 256 Hz
(e) 484 Hz

4. The time period of a simple pendulum of length L is 3 s. If the length of the pendulum is increased 4 times, the increase in time period of the simple pendulum is
(a) 3 s (b) 4 s
(c) 5 s (d) 6 s
(e) 2 s

35. The incorrect statement is
(a) The separation between two successive nodes is λ.
(b) Antinodes are formed at both ends of an open organ pipe.
(c) In a one end closed organ pipe node is formed at the closed end.
(d) Nodes are formed at both ends of stretched string.
(e) The separation between the successive node and antinode is $\lambda/4$.

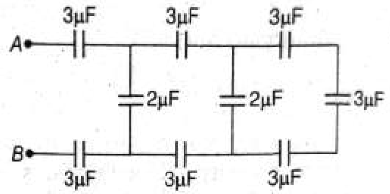
36. The ratio of the magnitudes of maximum acceleration to the corresponding velocity of a body undergoing simple harmonic motion, $x = a\sin 2\pi f t$ is
(a) $2\pi fa$ (b) $4\pi^2 fa$
(c) $2\pi f$ (d) infinity
(e) zero

37. The force experienced by a proton moving in an electric field of intensity $3E$ is (e is the charge of the electron)
(a) Ee (b) $2\,Ee$
(c) $3\,Ee$ (d) $Ee/2$
(e) $Ee/3$

38. Around a stationary charge of $+5\mu C$, another charge $-5\mu C$ is taken once round a circle of radius 4 cm. The amount of work done in joule is
(a) $\frac{2\pi}{5}$ (b) $\frac{3\pi}{8}$
(c) zero (d) $\frac{4\pi}{5}$
(e) $\frac{\pi}{4}$

39. The charge present in a doubly ionised helium atom is
(a) 1.6×10^{-19} C (b) 6.4×10^{-19} C
(c) 4.8×10^{-19} C (d) 8.0×10^{-19} C
(e) 3.2×10^{-19} C

40. The effective capacitance between A and B in the given figure is



(a) 1.5 μF (b) 1μF (c) 3 μF (d) 2 μF
(e) 2.5 μF

41. The electrostatic force between a proton and an electron for certain distance of separation is F_1 and that between an electron and positron at the same distance of separation is F_2. Then, the ratio $F_1 : F_2$ is
(a) 1 : 1 (b) 1 : 2 (c) 1879 : 1 (d) 1 : 1879
(e) 2 : 1

42. Conservation of charge and conservation of energy are respectively the basis of
(a) Joule's law and Ampere's circuital law
(b) Gauss' law and Ohm's law
(c) Kirchhoff's junction rule and loop rule
(d) Coulomb's inverse square law and Gauss' law
(e) Joule's law and Ohm's law

43. The incorrect statement is
(a) Resistivity of copper increases with increase of temperature.
(b) Resistivity of germanium decreases with the increase of temperature.
(c) Resistivity of semiconductors is higher than that of the conductors.
(d) Resistivity of nichrome shows a weak dependence with temperature.
(e) Resistivity of insulators is independent of temperature.

44. The three colours in a carbon resistor are red, black and orange. If the fourth colour is absent, then the value of tolerance of the resistor is
(a) ± 2000 Ω (b) ± 1000 Ω
(c) ± 3000 Ω (d) ± 4000 Ω
(e) ± 200 Ω

45. Material that is widely used to make wire bound standard resistors is
(a) manganin (b) iron
(c) copper (d) tungsten
(e) germanium

46. An electron and a proton moving with same velocity v enter into a uniform perpendicular magnetic field. Then,

(a) proton alone moves in straight line path
(b) electron alone moves in straight line path
(c) both move in straight line paths
(d) both move in elliptical paths
(e) both move in circular paths

47. In a moving coil galvanometer, when the number of turns of the coil is doubled,

(a) both the current sensitivity and voltage sensitivity are doubled
(b) the current sensitivity is halved but voltage sensitivity remains unchanged
(c) the current sensitivity remains unchanged but voltage sensitivity is doubled
(d) the current sensitivity is doubled but voltage sensitivity remains unchanged
(e) both the current sensitivity and voltage sensitivity remain unchanged

48. The strength of earth's magnetic field at a point is 0.4×10^{-5} T. If this field is to be annulled by the magnetic induction produced at the centre of a circular conducting loop of radius π cm, the current to be sent through the loop is

(a) 2 A (b) 0.15 A (c) 1.5 A (d) 0.2 A
(e) 1 A

49. Similar or same magnetic fields can be produced by

(a) a solenoid and a bar magnet
(b) a solenoid and a toroid
(c) a solenoid and a circular coil
(d) a circular coil and a toroid
(e) a bar magnet and a toroid

50. The incorrect statement is

(a) The direction of eddy currents is given by Lenz's law.
(b) A choke coil is a pure inductor used for controlling current in an AC circuit.
(c) The rms value of AC current is $\sqrt{2}$ times the peak value of AC current.
(d) Quality factor is a measure of sharpness of resonance in AC circuit.
(e) Magnetic field energy stored in an inductor of inductance L is $\frac{1}{2}LI^2$.

51. The ratio of energy stored per unit volume in a solenoid having magnetic induction B to the electrostatic energy stored per unit volume in a capacitor in electric field E is

(a) $\frac{B^2c}{E^2}$ (b) $\frac{B^2c^2}{E^2}$ (c) $\frac{Bc^2}{E^2}$ (d) $\frac{B^2c^2}{E}$
(e) $\frac{B^2c^2}{2E^2}$

52. Find the mismatch pair.

(a) Induction furnace : eddy current
(b) AC generator : armature coil
(c) L-C-R circuit : resonance
(d) Transformer : DC voltage
(e) Magnetic brakes : magnetic flux

53. When an AC voltage of $V = 330 \sin (100\pi t)$ is applied to a capacitor, it produces a current of $I = 1.5 \cos (100\pi t)$. The capacitive reactance of the capacitor is

(a) 120 Ω (b) 180 Ω (c) 200 Ω (d) 220 Ω
(e) 280 Ω

54. Radio waves are

(a) produced by hot bodies
(b) in the frequency range 10^9 Hz to 10^{12} Hz
(c) suitable for RADAR systems
(d) used in cellular phones to transmit voice communication
(e) used to kill germs in water purifiers

55. The electromagnetic waves that cause greenhouse effect are

(a) X-rays (b) cathode rays
(c) UV rays (d) gamma rays
(e) infrared rays

56. The power of a corrective lens is − 4.0 D. The lens is

(a) convex lens of focal length + 25 cm
(b) concave lens of focal length − 25 cm
(c) convex lens of focal length + 4 cm
(d) concave lens of focal length − 4 cm
(e) convex lens of focal length + 20 cm

57. The incorrect statement is

(a) Optical density is the ratio of speed of light in two media.
(b) Hotter air is less dense than the cooler air.
(c) Cooler air has higher refractive index than the hotter air.

(d) the refractive index of air decreases with its density.
(e) optical density of air increases with height of air layer.

58. A plane wave front is incident on a thin prism, thin convex lens and a concave mirror separately. The wave front(s) emerging out from the
(a) concave mirror is plane
(b) thin prism is spherical
(c) convex lens and concave mirror are plane
(d) convex lens and prism are plane
(e) convex lens and concave mirror are spherical

59. If the Young's double slit experimental set up is immersed in a liquid of refractive index μ, the fringe width of the interference pattern observed is β. When the experiment is performed in air medium with the same experimental set up, the fringe width of the pattern will be
(a) β (b) $\frac{\beta}{\mu}$ (c) $(\mu + 1)\beta$ (d) $\mu\beta$
(e) $(\mu - 1)\beta$

60. Two rays of light A and B are falling on a glass slab at the angles of incidence 45° and 60°. If the reflected ray of A is partially polarised and that of B is completely polarised, then the refractive index of glass is
(a) 1.33 (b) 1.414 (c) 1.5 (d) 1.65
(e) 1.732

61. The momenta of a proton, a neutron and an electron are in the ratio 3:2:1, then their respective de-Broglie wavelengths are in the ratio
(a) 1 : 1 : 1 (b) 2 : 3 : 6
(c) 1 : 2 : 3 (d) 6 : 3 : 2
(e) 4 : 2 : 1

62. The material that is not photo sensitive to visible light is
(a) caesium (b) sodium
(c) rubidium (d) cadmium
(e) potassium

63. The energy equivalent of 5g of a substance is
(a) 4.5×10^{12} J (b) 9×10^{12} J
(c) 4.5×10^{14} J (d) 4.5×10^{16} J
(e) 9×10^{16} J

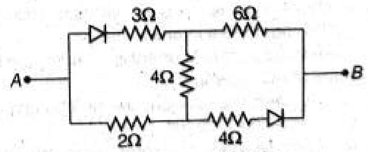
64. The incorrect statement is
(a) Nuclear density is independent of the mass number A of the nucleus.
(b) Average binding energy per nucleon is very high for light nuclei.
(c) Nuclear forces are strongest in nature.
(d) In a radioactive nucleus, the half life period is directly proportional to mean life.
(e) Becquerel (Bq) is the SI unit of activity of a radioactive source.

65. In Bohr atom model, the total energy of the electron in hydrogen atom is −3.4 eV. Then its angular momentum about the nucleus of the atom is (h = Planck's constant)
(a) $\frac{h}{\pi}$ (b) $\frac{h}{2\pi}$
(c) $\frac{2h}{\pi}$ (d) $\frac{4h}{\pi}$
(e) $\frac{h}{4\pi}$

66. In a nuclear reactor, the ratio of number of fission produced by a given generation of neutrons to the number of fission of the preceding generation is known as
(a) quality factor
(b) nuclear reaction factor
(c) multiplication factor
(d) fission ratio
(e) response ratio

67. The special purpose diode operated/ working under forward bias is / are
(a) Zener diode and LED
(b) photodiode and LED
(c) Zener diode and solar cell
(d) LED
(e) photo diode

68. If the potential at A is greater than the potential at B, then the equivalent resistance of the circuit across AB is



(a) 4.4 Ω (b) 5.2 Ω
(c) 6 Ω (d) 9 Ω
(e) 3.6 Ω

69. When light falls on a solar cell, the generation of emf happens due to
(a) generation of electron-hole pairs only
(b) generation and collection of electron-hole pairs only
(c) collection of electron-hole pairs only
(d) generation, separation and collection of electron-hole pairs
(e) separation and collection of electron-hole pairs only

70. Logic gates are given the inputs $A = 0$ and $B = 1$ in case (a) and $A = 1$ and $B = 0$ in case (b). The gates giving output of $y = 1$ for both the cases are
(a) OR and AND (b) OR and NAND
(c) AND and NOR (d) NOR and NAND
(e) AND and NAND

71. The minimum length of the dipole antenna for a carrier wave frequency of 200 MHz is nearly
(a) 1.75 m (b) 0.52 m (c) 0.25 m (d) 0.38 m
(e) 0.75 m

72. In communication systems, the device used to convert energy from one form to another form is
(a) repeater (b) transducer
(c) amplifier (d) attenuator
(e) antenna

Chemistry

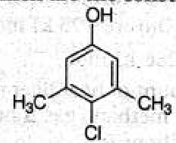
73. Which one of the following contains the highest number of oxygen atoms?
(a) One mole of aluminium sulphate
(b) Two moles of ferrous sulphate
(c) Three moles of hydrogen peroxide
(d) Two moles of potassium permanganate
(e) One mole of potassium dichromate

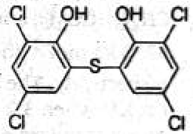
74. Among the following pairs of compounds, the one that does not illustrate the law of multiple proportions, is
(a) NO and NO_2 (b) CuO and Cu_2O
(c) FeO and Fe_2O_3 (d) H_2O and H_2S
(e) NO and N_2O

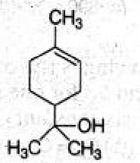
75. A dinegative ion of the element X consists of 10 electrons and 8 neutrons. A dipositive ion of the element Y consists of 12 protons. The number of neutrons in Y is 1.5 times the number of electrons in atoms X. Then the mass numbers of X and Y would be in the ratio
(a) 1 : 2 (b) 2 : 3 (c) 3 : 2 (d) 2 : 5
(e) 1 : 3

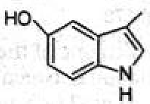
76. A particle of mass 6.6×10^{-31} kg is moving with a velocity of 1×10^{7} ms^{-1}. The de-Broglie wavelength (in Å) associated with the particle, is ($h = 6.6 \times 10^{-34}$ Js)
(a) 1 (b) 10 (c) 5 (d) 2
(e) 4

77. From the following, choose the correct structures of chloroxylenol and terpineol, which are the constituents of "dettol"?

I. 

II. 

III. 

IV. 

(a) I and II (b) II and III (c) I and IV (d) I and III
(e) II and IV

78. A fast moving particle of mass 6.63×10^{-28} g can be located with an accuracy of 1Å. The uncertainty in its velocity (in ms^{-1}) is about ($h = 6.63 \times 10^{-34}$ Js)
(a) 8×10^{3} (b) 8×10^{4} (c) 8×10^{5} (d) 8×10^{6}
(e) 8×10^{7}

79. Which one of the following molecules contains an incomplete octet of the central atom?
(a) SF_6 (b) $AlCl_3$ (c) CH_4 (d) PF_5
(e) H_2O

80. Which one of the following reactions involves change from sp^2 to sp^3-hybridisation of the central atom?

(a) $CH_4 + 2Cl_2 \longrightarrow CH_2Cl_2 + 2HCl$
(b) $NH_3 + H^+ \longrightarrow NH_4^+$
(c) $AlCl_3 + Cl^- \longrightarrow AlCl_4^-$
(d) $H_2O + H^+ \longrightarrow H_3O^+$
(e) $PCl_3 + Cl_2 \longrightarrow PCl_5$

81. The dipole-dipole interaction energy between rotating polar molecules is proportional to ...., where 'r' is the distance between polar molecules.

(a) $1/r^4$ (b) $1/r^9$ (c) $1/r^3$ (d) $1/r^2$
(e) $1/r^6$

82. A metal 'X' crystallises in a body-centered cubic structure and its metallic radius is 346.4 pm. The length (in pm) of the unit cell is

(a) 200 (b) 800 (c) 600 (d) 500
(e) 400

83. The standard enthalpy of formation of $CH_4(g)$, $CO_2(g)$ and $H_2O(l)$ are -75 kJ mol^{-1}, -393 kJ mol^{-1} and -286 kJ mol^{-1}, respectively. The amount of heat liberated (in kJ) when 3.2 g of methane gas is burnt under standard conditions is

(a) 89 (b) 278 (c) 890 (d) 965
(e) 178

84. Which one of the following is the correct relation between C_p and C_V for one mole of an ideal gas? (R is molar gas constant)

(a) $C_p = C_V - R$ (b) $C_p = C_V + R$
(c) $C_p = R - C_V$ (d) $C_p = C_V \times R$
(e) $C_p = C_V / R$

85. Some of the reactions and their equilibrium constants K_C are given. Choose the reaction which proceeds rarely at the given temperature.

(a) $2H_2(g) + O_2(g) \rightleftharpoons 2H_2O(g)$; $K_C = 2.4 \times 10^{47}$ at 500 K
(b) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$; $K_C = 57.0$ at 700 K
(c) $H_2(g) + Cl_2(g) \rightleftharpoons 2HCl(g)$; $K_C = 4.0 \times 10^{31}$ at 300 K
(d) $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$; $K_C = 4.8 \times 10^{-31}$ at 298 K
(e) $H_2(g) + Br_2(g) \rightleftharpoons 2HBr(g)$; $K_C = 5.4 \times 10^{18}$ at 300 K

86. The equilibrium constants for the following two reactions at 298 K are given below :

$$2A \rightleftharpoons B + C;\ K_1 = 16$$
$$2B + C \rightleftharpoons 2X;\ K_2 = 25$$

What is the value of K for the reaction, $A + \frac{1}{2}B \rightleftharpoons X$ at 298K?

(a) 1/5 (b) 1/40 (c) 5/4 (d) 4/5
(e) 20

87. The average oxidation number of bromine in Br_3O_8 is

(a) 16/3 (b) 4/3 (c) 3/6 (d) 5/2
(e) 8/3

88. The standard electrode potentials of some electrodes are given below:

Fe^{3+}/Fe^{2+} 0.77V; Br_2/Br^- 1.09 V;
I_2/I^- 0.54V; $Zn^{2+}/Zn(s)$ -0.76 V;
$Ag^+/Ag(s)$ 0.80 V; $Fe^{2+}/Fe(s)$ -0.44 V;
$Cu^{2+}/Cu(s)$ 0.34 V

Predict the reaction that is not feasible.

(a) Fe^{3+} (aq) oxidises $I^-(aq)$
(b) $Ag^+(aq)$ oxidises $Cu(s)$
(c) $Ag(s)$ reduces Fe^{3+} (aq)
(d) $Br_2(aq)$ oxidises Fe^{2+} (aq)
(e) $Zn(s)$ reduces Cu^{2+} (aq)

89. The chemistry teacher asked the students to prepare 20% w/w solution of urea $[NH_2CONH_2]$ in water. Which one of the following solution does not conform to the required composition?

(a) 6 g urea dissolved in 24 g water
(b) 20 g urea dissolved in 80 g water
(c) 10 g urea dissolved in 40 g water
(d) 4 g urea dissolved in 16 g water
(e) 15 g urea dissolved in 30 g water

90. The vapour pressure of pure liquids X and Y at 350 K are 200 mm and 300 mm of Hg, respectively. Then the correct vapour pressure (in mm of Hg) of an ideal solution containing X and Y in the mole ratio 3 : 2 at the same temperature is

(a) 120 (b) 180 (c) 260 (d) 240
(e) 160

91. In a reaction $3A \longrightarrow$ products, the concentration of A decreases from 0.4 mol L^{-1} to 0.1 mol L^{-1} in 20 min at 300 K. The rate of decrease in $[A]$ during this interval (in $\text{mol L}^{-1}\text{min}^{-1}$) at 300 K is

(a) 0.005 (b) 0.015 (c) 0.001 (d) 0.15
(e) 0.05

92. The half-life period of a first order reaction at 298K is 20 min. The time (in min.) required for 99.9% completion of the reaction at the same temperature, is

(a) 100 (b) 200 (c) 150 (d) 250
(e) 300

93. The critical temperature of some gases are methane 190K, ammonia 405K, carbon dioxide 304K, *n*-butane 425K and dihydrogen 33K. The gas that is adsorbed to the maximum extent on 1 g of activated charcoal at a given temperature is

(a) dihydrogen (b) methane
(c) carbon dioxide (d) *n*-butane
(e) ammonia

94. Which one of the following is not true with regard to physisorption?

(a) It arises because of van der Waals' forces.
(b) It is not specific in nature.
(c) High activation energy is needed.
(d) It depends on the nature of gas.
(e) Enthalpy of adsorption is low (20-40 kJ mol^{-1}).

95. Match the following.

A.	Saline hydride	(i)	CrH
B.	Electron-deficient hydride	(ii)	CH_4
C.	Electron-precise hydride	(iii)	BeH_2
D.	Electron-rich hydride	(iv)	B_2H_6
E.	Metallic hydride	(v)	H_2O

Choose the correct option.

(a) A-(iii); B-(ii); C-(iv); D-(v); E-(i)
(b) A-(iii); B-(v); C-(iv); D-(ii); E-(i)
(c) A-(iv); B-(ii); C-(iii); D-(v); E-(i)
(d) A-(iii); B-(iv); C-(ii); D-(v); E-(i)
(e) A-(iii); B-(i); C-(ii); D-(iv); E-(v)

96. The metal which dissolves in liquid ammonia to give a blue-black solution due to formation of solvated electron is

(a) aluminium (b) gallium
(c) calcium (d) silicon
(e) germanium

97. Which one of the following processes does not produce dinitrogen?

(a) Thermal decomposition of ammonium dichromate
(b) Thermal decomposition of barium azide
(c) Treating an aqueous solution of ammonium chloride with sodium nitrite
(d) Thermal decomposition of sodium azide
(e) Thermal decomposition of ammonium nitrate

98. Which of the following compounds is used as refrigerant?

(a) CCl_2F_2 (b) $ClCH_2CH_2SCH_2CH_2Cl$
(c) CCl_4 (d) CCl_3NO_2
(e) $COCl_2$

99. Which of the following set of transition metals have high volatility?

(a) Ti, Zn and Hf
(b) Cr, Mo and W
(c) Mn, Tc and Re
(d) Fe, Ru and Os
(e) Zn, Cd and Hg

100. Both Cr^{2+} and Mn^{3+} have d^4 configuration. Which one of the following is true?

(a) Mn^{3+} is a reducing agent but Cr^{2+} is an oxidising agent.
(b) Mn^{3+} is an oxidising agent but Cr^{2+} is a reducing agent.
(c) Both Mn^{3+} and Cr^{2+} are oxidising agents.
(d) Both Mn^{3+} and Cr^{2+} are reducing agents.
(e) Both Mn^{3+} and Cr^{2+} are neither reducing nor oxidising agents.

101. The complexes $[Co(NH_3)_5NO_2]Cl_2$ and $[Co(NH_3)_5ONO]Cl_2$ are

(a) coordination isomers
(b) geometrical isomers
(c) solvate isomers
(d) ionisation isomers
(e) linkage isomers

102. Which one of the following is not an ore of iron?
(a) Magnesite (b) Haematite
(c) Magnetite (d) Siderite
(e) Iron pyrites

103. The overall complex dissociation equilibrium constant for $[Cr(H_2O)_6]^{3+}$ ion is 5×10^{-12}. The overall stability constant of the complex is
(a) 2×10^{-11} (b) 5×10^{11} (c) 5×10^{10} (d) 2×10^{11}
(e) 0.2×10^{11}

104. Match the following.

A.	Alkane	(i)	Phenol
B.	Alicyclic compound	(ii)	Tropolone
C.	Benzenoid aromatic compound	(iii)	Isobutane
D.	Non-benzenoid aromatic compound	(iv)	Furan
E.	Heterocyclic compound	(v)	Cyclohexene

Choose the correct option.
(a) A-(iii); B-(i); C-(v); D-(ii); E-(iv)
(b) A-(iii); B-(v); C-(i); D-(ii); E-(iv)
(c) A-(i); B-(ii); C-(iii); D-(iv); E-(v)
(d) A-(iii); B-(v); C-(i); D-(iv); E-(ii)
(e) A-(iii); B-(ii); C-(i); D-(v); E-(iv)

105. The elemental analysis of an organic compound gave C: 38.71%, H : 9.67%. What is the empirical formula of the compound?
(a) CH_2O (b) CH_3O (c) CH_4O (d) CHO
(e) CH_5O

106. Which one of the following molecules contains only primary and tertiary carbon atoms?
(a) 2, 2-dimethylbutane (b) 3-methylpentane
(c) 2, 3-dimethylbutane (d) *n*-hexane
(e) 2-methylhexane

107. Calculate the number of σ and π-bonds in 2-*n*-propylpent-1-ene.
(a) 22 σ-bonds, 2π-bonds
(b) 23 σ-bonds, 1π-bond
(c) 21 σ-bonds, 1π-bond
(d) 23 σ-bonds, 2π-bonds
(e) 20 σ-bonds, 1π-bond

108. Which one of the following molecules gives four isomeric monochlorides on photochemical chlorination?
(a) 2-methylpropane (b) *n*-butane
(c) 2-methylbutane (d) 2, 3-dimethylbutane
(e) Propane

109. Which of the following aryl chlorides on warming with water forms the corresponding phenol?
(a) 4-methylchlorobenzene
(b) 4-nitrochlorobenzene
(c) 2, 4, 6-trinitrochlorobenzene
(d) 2-nitrochlorobenzene
(e) 2, 4-dinitrochlorobenzene

110. Resorcinol is
(a) benzene-1, 3-diol (b) benzene-1, 4-diol
(c) benzene-1, 2-diol (d) 3-methylphenol
(e) 4-methylphenol

111. Choose the correct order of acidity of the following phenols:
I. *m*-nitrophenol II. *p*-cresol
III. *p*-nitrophenol IV. phenol
(a) III > I > IV > II (b) II > IV > III > I
(c) I > II > III > IV (d) IV > II > III > I
(e) III > II > I> IV

112. Which one of the following represents valeraldehyde?
(a) $CH_3CH_2CH_2CH_2CHO$
(b) $CH_3CH(CH_3)CH_2CHO$
(c) $CH_3CH(OCH_3)CHO$
(d) $(CH_3)_2CHCHO$
(e) $CH_3CH_2CH(CH_3)CHO$

113. Toluene on treatment with chromic oxide in acetic anhydride at 273 K to 283K gives
(a) benzaldehyde (b) benzylidene diacetate
(c) benzoic acid (d) benzyl alcohol
(e) phenylacetate

114. Among methanamine, ethanamine, benzenamine, N-methylaniline and N, N-dimethylaniline, the weakest and the strongest base in aqueous phase, respectively are
(a) benzenamine and methanamine
(b) N-methylaniline and ethanamine
(c) N, N-dimethylaniline and ethanamine
(d) benzenamine and ethanamine
(e) N-methylaniline and methanamine

115. The product formed, when benzene diazonium fluoroborate is heated with aqueous sodium nitrite solution in the presence of copper is
(a) fluorobenzene (b) benzene
(c) phenol (d) *p*-nitrophenol
(e) nitrobenzene

116. Which one of the following is a polysaccharide?
(a) Glycogen (b) Lactose
(c) Maltose (d) Sucrose
(e) Glucose

117. Which of the following is added to commercial salt to control hypothyroidism?
(a) Magnesium iodide (b) Potassium iodide
(c) Sodium iodide (d) Calcium iodide
(e) Lithium iodide

118. Conveyor belt is manufactured from
(a) buna-S (b) neoprene
(c) PVC (d) teflon
(e) glyptal

119. Which one of the following is a non-narcotic analgesics?
(a) Morphine (b) Codeine
(c) Paracetamol (d) Heroin
(e) Bithional

120. The primary precursor of photochemical smog that can be metabolised by plants such as juniparus and pyrus, is
(a) nitrogen dioxide
(b) ozone
(c) PAN
(d) carbon dioxide
(e) sulphur dioxide

Mathematics

1. Let $A = \{1, 2, 3, 4, 5\}$ and let $B = \{1, 2, 3, 4\}$. If the relation $R : A \to B$ is given by $(a, b) \in R$ if and only if $a + b$ is even, then $n(R)$ is equal to
(a) 10 (b) 16 (c) 20 (d) 12
(e) 6

2. The domain of the function $f(x) = (x^2 - 2x - 63)^{3/2}, x \in R$ is
(a) $(-\infty, -6] \cup [9, \infty)$ (b) $(-\infty, -9] \cup (7, \infty)$
(c) $(-\infty, -7] \cup [7, \infty)$ (d) $(-\infty, -5] \cup [9, \infty)$
(e) $(-\infty, -7] \cup [9, \infty)$

3. Let $A = \{x \in Z : -1 \le x < 4\}$ and let $B = \{x \in Z : 0 < \frac{x}{2} \le 3\}$. Then, $A \cap B$ is equal to
(a) $\{1, 2, 3\}$ (b) $\{2, 3\}$
(c) $\{1, 2, 3, 4\}$ (d) $\{2, 3, 4\}$
(e) $\{0, 1, 2, 3\}$

4. Let $f(x) = \begin{cases} x + 2, & \text{for } x < 1 \\ 4x - 1, & \text{for } 1 \le x \le 3 \\ x^2 + 5, & \text{for } x > 3 \end{cases}$. Then,
(a) $f(x)$ is not continuous at $x = -1$
(b) $f(x)$ is continuous at $x = 1$
(c) $f(x)$ is continuous at $x = 3$
(d) $f(x)$ is not continuous at $x = 5$
(e) $f(x)$ is not continuous at $x = 2$

5. Let $\odot$ be a binary operation on $Q - \{0\}$ defined by $a \odot b = \frac{a}{b}$. Then, $1 \odot (2 \odot (3 \odot 4))$ is equal to
(a) 3/2 (b) 8/3 (c) 4/3 (d) 3/4
(e) 3/8

6. Let $f : R \to R$ be defined by $f(x) = \cos x$. Then,
(a) f is one-one and odd
(b) f is odd but not one - one
(c) f is even and onto
(d) f is one - one and even
(e) f is even but not onto

7. If $n(A \cup B) = 97, n(A \cap B) = 23$ and $n(A - B) = 39$, then $n(B)$ is equal to
(a) 52 (b) 55 (c) 58 (d) 62
(e) 65

8. The principal argument of the complex number $z = \frac{8 + 4i}{1 + 3i}$ is equal to
(a) $\frac{\pi}{4}$ (b) $\frac{-\pi}{4}$
(c) $\frac{3\pi}{4}$ (d) $\frac{-3\pi}{4}$
(e) $\frac{\pi}{6}$

9. The minimum value of $|z+1|+|z-2|$ is equal to
(a) 1 (b) 2 (c) 3 (d) 4
(e) 0

10. If $z = \dfrac{(3+i)(7-i)^2}{3-i}$, then the value of $|z|$ is equal to
(a) 48 (b) $\sqrt{50}$ (c) 50 (d) $\sqrt{500}$
(e) $\sqrt{48}$

11. The value of $\left[\dfrac{5i}{(1-i)(2-i)(3-i)}\right]^{50}$ is equal to
(a) $\left(\frac{1}{2}\right)^{25}$ (b) $\left(\frac{1}{2}\right)^{50}$
(c) $-\left(\frac{1}{2}\right)^{25}$ (d) $-\left(\frac{1}{2}\right)^{50}$
(e) $\left(\frac{1}{10}\right)^{50}$

12. If $z^4 = 7-5i$, then $\text{Im}\,((\bar{z})^4)$ is equal to
(a) 5 (b) 7 (c) −7 (d) −5
(e) 0

13. The modulus of $\left(\dfrac{1+i}{1-i}\right)^{75} - \left(\dfrac{1-i}{1+i}\right)^{75}$ is
(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 4
(e) 16

14. If z_1 and z_2 are two different complex numbers with $|z_2| = 1$, then $\left|\dfrac{1-\bar{z}_1 z_2}{z_1 - z_2}\right|$ is equal to
(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$
(e) 1

15. If $-1+7i, -1+xi$ and $3+3i$ are the three vertices of an isosceles triangle which is right angled at $-1+xi$, then the value of x is equal to
(a) −1 (b) 3 (c) −3 (d) 7
(e) −7

16. The sum of the first 24 terms of the series $9+13+17+\ldots$ is equal to
(a) 1212 (b) 1200 (c) 1440 (d) 1320
(e) 1230

17. In an AP there are 18 terms and the last three terms of the AP are 67, 72, 77. Then, the first term of the AP is
(a) −7 (b) 9 (c) −9 (d) −8
(e) 7

18. If the first term of a GP is 3 and the sum of second and third terms is 60, then the common ratio of the GP is
(a) 4 or −3 (b) 4 only
(c) 4 or 5 (d) 4 or −5
(e) −5 only

19. If nth term of a series is $n+(-1)^{n-1}$, $n = 1, 2, 3 \ldots$, then the sum of first 40 terms of the series is
(a) 810 (b) 820 (c) 821 (d) 819
(e) 780

20. The 11th term of the geometric series $\sum_{r=0}^{20} 2\times(-2)^r$ is equal to
(a) −4096 (b) 1024 (c) 2048 (d) 1048
(e) −2024

21. Let S_n be the sum of the first n terms of the series $a_1 + a_2 + \ldots a_n + \ldots$. If $S_n = n^2 + 4n$, then the nth term a_n is
(a) $2n+3$ (b) $2n-1$ (c) $2n+5$ (d) $2n-3$
(e) $2n$

22. Let $t_n = \dfrac{1}{n}\sum_{k=1}^{n}\left(\dfrac{k}{n}\right)^2$ for $n = 1, 2, 3, \ldots$ Then, t_{10} is equal to
(a) $\frac{7}{600}$ (b) $\frac{231}{100}$ (c) $\frac{209}{600}$ (d) $\frac{11}{200}$
(e) $\frac{77}{200}$

23. The number of arrangements containing all the seven letter of the word ALRIGHT that begins with LG is
(a) 720 (b) 120 (c) 600 (d) 540
(e) 760

24. The number of numbers greater than 6000 that can be formed from the digits 3, 5, 6, 7 and 9 (no digit is repeated in a number) is equal to
(a) 264 (b) 720 (c) 192 (d) 132
(e) 544

25. The number of subsets containing exactly 4 elements of the set { 2, 4, 6, 8, 10, 12, 14, 16, 18} is equal to
(a) 126 (b) 63 (c) 189 (d) 58
(e) 94

26. If ${}^{11}P_r = 7920$ and ${}^{11}C_r = 330$, then the value of r is equal to
(a) 2 (b) 3 (c) 4 (d) 5
(e) 6

27. In the binomial expansion of $(x - 2y^2)^9$, the coefficient of $x^6 y^6$ is equal to
(a) –672 (b) 672
(c) 336 (d) –336
(e) –512

28. Let $(3+x)^{10} = a_0 + a_1(1+x) + a_2(1+x)^2 + \dots a_{10}(1+x)^{10}$, where $a_1, a_2 \dots a_{10}$ are constants. Then, the value of $a_0 + a_1 + a_2 + \dots a_{10}$ is equal to
(a) 2^{20} (b) 2^{10} (c) 3^{10} (d) 2^{11}
(e) 2^{15}

29. If ${}^nC_5 + {}^nC_6 = {}^{51}C_6$, then the value of n is equal to
(a) 49 (b) 50 (c) 45 (d) 46
(e) 51

30. Let $A = \begin{bmatrix} 3 & 4 \\ 1 & -2 \end{bmatrix}$ and let $AB = \begin{bmatrix} -5 & 41 \\ 5 & -13 \end{bmatrix}$.
Then, $|B^T| =$
(a) $\frac{1}{14}$ (b) 14 (c) 10 (d) –10
(e) –14

31. Let $A = \begin{vmatrix} 2 & 1 & -2 \\ 1 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix}$ and let $B = |A| \operatorname{adj}(A)$.
Then, $|B| =$
(a) 256 (b) 64 (c) 512 (d) 1024
(e) 128

32. The values of x satisfying the equation
$\begin{vmatrix} x & 4 & 0 \\ 2 & 2 & -x \\ 1 & 1 & 1 \end{vmatrix} = 0$ are
(a) 2, – 4 (b) 1, 2 (c) – 1, 2 (d) –1, –2
(e) –2, 4

33. If $A = [2\ 0\ 6]$ and $B = \begin{bmatrix} 3 & 5 \\ 7 & -2 \\ 6 & 6 \end{bmatrix}$, then $AB =$
(a) [42 46] (b) $\begin{bmatrix} 42 \\ 46 \end{bmatrix}$
(c) $\begin{bmatrix} 6 & 10 \\ 0 & 0 \\ 36 & 36 \end{bmatrix}$ (d) [17 19]
(e) $\begin{bmatrix} 2 & 12 \\ 14 & -4 \end{bmatrix}$

34. If A is non-singular matrix and if $A^{-1} = \frac{1}{2}\begin{bmatrix} -10 & -4 \\ 2 & 1 \end{bmatrix}$, then adj $(A) =$
(a) $\begin{bmatrix} -1 & -4 \\ 2 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & 4 \\ -2 & -1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 4 \\ -2 & -10 \end{bmatrix}$ (d) $\begin{bmatrix} -10 & -4 \\ 2 & 1 \end{bmatrix}$
(e) $\begin{bmatrix} -1 & -4 \\ 10 & 2 \end{bmatrix}$

35. $\begin{vmatrix} \sin\alpha & \cos(\alpha+\theta) & \cos\alpha \\ \sin\beta & \cos(\beta+\theta) & \cos\beta \\ \sin\gamma & \cos(\gamma+\theta) & \cos\gamma \end{vmatrix} =$
(a) – 1 (b) 1 (c) 2 (d) 4
(e) 0

36. The solution set of the inequality $-2 \le \frac{3x+2}{2} < 7$ is
(a) $\{x : 3 \le x < 4\}$ (b) $\{x : -2 \le x < 3\}$
(c) $\{x : -2 \le x < 4\}$ (d) $\{x : 0 \le x < 6\}$
(e) $\{x : -2 \le x < 6\}$

37. The set of all x satisfying the inequality $|3x + 4| \le 7$ is
(a) $\left[-1, \frac{11}{3}\right]$ (b) $\left[\frac{4}{3}, \frac{7}{3}\right]$ (c) $\left[\frac{-11}{3}, 1\right]$ (d) $\left[\frac{-4}{3}, \frac{7}{3}\right]$
(e) $\left[\frac{-4}{3}, \frac{11}{3}\right]$

38. If the solution set of the inequality $|a + 3x| \le 6$ is $\left[\frac{-8}{3}, \frac{4}{3}\right]$, then the value of a is equal to
(a) –1 (b) –2 (c) 4 (d) –4
(e) 2

39. Consider the following statements:

(i) For every positive real number x, $x-10$ is positive.

(ii) Let n be a natural number. If n^2 is even, then n is even.

(iii) If a natural number is odd, then its square is also odd.

Then,

(a) (i) False, (ii) True and (iii) True
(b) (i) False, (ii) False and (iii) True
(c) (i) True, (ii) False and (iii) True
(d) (i) True, (ii) True and (iii) True
(e) (i) False, (ii) True and (iii) False

40. If $\cos\theta = \frac{5}{11}$ and $\tan\theta < 0$, then the value of $\sin\theta$ is equal to

(a) $\frac{8\sqrt{6}}{11}$ (b) $\frac{-8\sqrt{6}}{11}$ (c) $\frac{4\sqrt{6}}{11}$ (d) $\frac{-4\sqrt{6}}{11}$
(e) $\frac{6}{11}$

41. If α and β are two acute angles of a right triangle, then
$(\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2 =$

(a) $1+\sin 2\alpha$ (b) $2(1+\sin 2\alpha)$
(c) $1+\cos 2\alpha$ (d) $2(1+2\cos 2\alpha)$
(e) $2+\sin 2\alpha$

42. The range of the function $f(x) = 2\sin(3x)+1$ is equal to

(a) $[-1, 1]$ (b) $\left[\frac{-1}{3}, \frac{1}{3}\right]$ (c) $[-2, 1]$ (d) $[-1, 2]$
(e) $[-1, 3]$

43. The period of the function
$g(x) = 5\cot\left(\frac{\pi}{3}x + \frac{\pi}{6}\right) + 2$ is equal to

(a) 2 (b) 3 (c) 4 (d) 5
(e) 6

44. If $\theta \in (-\pi, 0)$ and $\cos\theta = \frac{-12}{13}$, then $\sin\left(\frac{\theta}{2}\right) =$

(a) $\frac{-5\sqrt{26}}{26}$ (b) $\frac{5\sqrt{26}}{26}$
(c) $\frac{-5\sqrt{13}}{13}$ (d) $\frac{5\sqrt{13}}{13}$
(e) $\frac{-5\sqrt{13}}{26}$

45. The solutions of the equation $\cos\theta = 2 - 3\sin(\theta/2)$ in the interval $0 \le \theta \le \pi$ are

(a) $\frac{\pi}{4}, \pi$ (b) $\frac{\pi}{3}, \frac{\pi}{2}$ (c) $\frac{\pi}{3}, \pi$ (d) $\frac{\pi}{6}, \frac{\pi}{2}$
(e) $\frac{\pi}{6}, \pi$

46. The value of $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$ is equal to

(a) $\frac{7\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$
(e) $\frac{5\pi}{6}$

47. The value of $\tan\left(\sin^{-1}\left(\frac{7}{25}\right)\right)$ is equal to

(a) $\frac{18}{25}$ (b) $\frac{24}{25}$ (c) $\frac{7}{24}$ (d) $\frac{3}{4}$
(e) $\frac{7}{18}$

48. $\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{200}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{200}\right)\right) =$

(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) 1
(e) 0

49. The equation of the straight line parallel to $y = -3x$ and passing through the point $(3, -2)$ is

(a) $y = -3x + 7$ (b) $y = -3x + 9$
(c) $y = -3x - 11$ (d) $y = -3x - 7$
(e) $y = -3x + 11$

50. The intercepts of a line with coordinate axes are equal. If the line passes through $(2, 3)$, then its equation is

(a) $2x + 3y = 5$ (b) $x + y = 5$
(c) $5x + 5y = 1$ (d) $x + y = 6$
(e) $3x + 2y = 5$

51. If the line $y = mx + c$ is perpendicular to $y = 1 + x$ and passes through the point $(1, 2)$, then the value of c is equal to

(a) 1 (b) −1 (c) −3 (d) 3
(e) 0

52. Let $A(-1, 2)$, $B(1, 3)$ and $C(a, b)$ be collinear. If B divides AC such that $BC = 8\,AB$, then the coordinates of C are

(a) $\left(\frac{5}{4}, \frac{25}{8}\right)$ (b) $(17, 9)$ (c) $(17, 11)$ (d) $\left(\frac{5}{4}, \frac{5}{8}\right)$
(e) $(1, 11)$

53. If the lines $2x-3y+5=0$, $9x-5y+14=0$ and $3x-7y+\lambda=0$ are concurrent, then the value of λ is equal to

(a) 7 (b) 8 (c) 10 (d) 9
(e) 6

54. The points of intersection of the line $y=x+2$ and the circle $(x-2)^2+y^2=16$ are

(a) $(-2,0),(2,4)$ (b) $(-2,4),(2,0)$
(c) $(4,0),(4,2)$ (d) $(4,6),(4,-2)$
(e) $(4,0),(4,-2)$

55. The three vertices of a triangle are $(0,0)$, $(3,1)$ and $(1,3)$. If this triangle is inscribed in a circle, then the equation of the circle is

(a) $2x^2+2y^2-2x-6y=0$
(b) $x^2+y^2-3x-y=0$
(c) $x^2+y^2-x-3y=0$
(d) $2x^2+2y^2-6x-2y=0$
(e) $2x^2+2y^2-5x-5y=0$

56. The equation of the circle touching the X-axis at $(5,0)$ and the line $y=10$ is

(a) $x^2+y^2-10x-10y+25=0$
(b) $x^2+y^2-10x-10y-25=0$
(c) $x^2+y^2-5x-5y-5=0$
(d) $x^2+y^2-5x-5y+5=0$
(e) $x^2+y^2+10x+10y-25=0$

57. If the radius of the circle $x^2+y^2+ax+by+3=0$ is 2, then the point (a,b) lies on the circle

(a) $x^2+y^2=7$ (b) $x^2+y^2=4$
(c) $x^2+y^2=14$ (d) $x^2+y^2=28$
(e) $x^2+y^2=1$

58. If the line $2x-3y+c=0$ passes through the focus of the parabola $x^2=-8y$, then the value of c is equal to

(a) 4 (b) -6 (c) 6 (d) -4
(e) 2

59. The centre of the ellipse $x^2+7y^2-14x+28y+49=0$ is

(a) $(7,0)$ (b) $(7,-4)$
(c) $(7,-2)$ (d) $(-7,4)$
(e) $(-7,2)$

60. The end points of the major axis of an ellipse are $(2,4)$ and $(2,-8)$. If the distance between foci of this ellipse is 4, then the equation of the ellipse is

(a) $\frac{(x-2)^2}{32}+\frac{(y+2)^2}{36}=1$
(b) $\frac{(x-4)^2}{32}+\frac{(y+2)^2}{36}=1$
(c) $\frac{(x-2)^2}{36}+\frac{(y+2)^2}{32}=1$
(d) $\frac{(x-2)^2}{32}+\frac{(y-4)^2}{36}=1$
(e) $\frac{(x-2)^2}{36}+\frac{(y-4)^2}{32}=1$

61. If $(-1,0)$ and $(3,0)$ are foci of an ellipse and the length of the major axis is 6, then the length of the minor axis is

(a) $\sqrt{5}$ (b) 5 (c) 10 (d) $2\sqrt{5}$
(e) 3

62. The eccentricity of the hyperbola $\frac{(x-3)^2}{9}-\frac{4(y-1)^2}{45}=1$ is equal to

(a) $\frac{3}{\sqrt{5}}$ (b) $\frac{5}{3}$ (c) $\frac{5}{\sqrt{3}}$ (d) $\frac{5}{2}$
(e) $\frac{3}{2}$

63. If $\mathbf{a}\times\mathbf{b}=7\hat{\mathbf{i}}+9\hat{\mathbf{j}}+10\hat{\mathbf{k}}$ and $\mathbf{a}\cdot\mathbf{b}=-20$, then $|\mathbf{a}|^2|\mathbf{b}|^2=$

(a) 530 (b) 580 (c) 400 (d) 630
(e) 560

64. Let $\mathbf{a}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and $\mathbf{a}+\mathbf{b}=4\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\lambda\hat{\mathbf{k}}$. If $\mathbf{a}\cdot\mathbf{b}=4$, then the value of λ is equal to

(a) 3 (b) -3 (c) -6 (d) 6
(e) 0

65. If $|\mathbf{a}|=\sqrt{14}$, $|\mathbf{b}|=\sqrt{10}$, $|\mathbf{a}-\mathbf{b}|=\sqrt{24}$ and θ is angle between $\mathbf{a}$ and $\mathbf{b}$, then $\cos\theta=$

(a) $\frac{\sqrt{35}}{70}$ (b) $\frac{\sqrt{6}}{12}$ (c) $\frac{\sqrt{15}}{60}$ (d) $\frac{\sqrt{210}}{35}$
(e) 0

66. If $|\mathbf{a}|=10$ and $|\mathbf{b}|=5$, then the value of $(\mathbf{a}+2\mathbf{b})\cdot(\mathbf{a}-2\mathbf{b})$ is equal to

(a) 32 (b) 16 (c) 8 (d) 4
(e) 0

67. If $\mathbf{a} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, then the value of $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b}$ is equal to

(a) 3 (b) −3 (c) 7 (d) −7
(e) 0

68. If **a** and **b** are position vectors of the points $(\alpha, 3, 0)$ and $(1, 0, 0)$ respectively and if the angle between the vectors **a** and **b** is $\frac{\pi}{4}$, then the value of α is equal to

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

69. If $\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, then a unit vector in the direction of **a** + **b** is

(a) $\frac{1}{6}(3\hat{i} + 6\hat{j} - 2\hat{k})$ (b) $\frac{1}{\sqrt{70}}(3\hat{i} + 6\hat{j} - 5\hat{k})$
(c) $\frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$ (d) $\frac{1}{\sqrt{50}}(3\hat{i} + 6\hat{j} - 3\hat{k})$
(e) $\frac{-1}{\sqrt{6}}(\hat{i} + 2\hat{j} - \hat{k})$

70. If $|\mathbf{u}| = 3, |\mathbf{v}| = 2$ and $|\mathbf{u} \times \mathbf{v}| = 3$, then the angle between **u** and **v** is equal to

(a) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ (b) $\frac{\pi}{6}$ or $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$ (d) $\frac{\pi}{2}$
(e) 0

71. The equation of the plane passing through the point $(-1, -2, -3)$ and perpendicular to the X-axis is

(a) $x = -1$ (b) $y = -2$
(c) $z = -3$ (d) $2x + 3y = 5$
(e) $x + y + z = 6$

72. Let L_1 be the line joining $(0, 0, 0)$ and $(1, 2, 3)$ and L_2 be the line joining $(2, 3, 4)$ and $(3, 4, 5)$. The point of intersection of L_1 and L_2 is

(a) (0, 0, 0) (b) (1, 2, 3) (c) (2, 3, 4) (d) (3, 4, 5)
(e) (1, 1, 1)

73. The equation of the line through the point $(1, -1, 1)$ and parallel to the line joining the points $(-2, 2, 0)$ and $(-1, 1, 1)$ is

(a) $\frac{x-1}{-3} = \frac{y-1}{-1} = z - 1$
(b) $1 - x = 1 + y = 1 - z$
(c) $x + 1 = -(y - 1) = z - 1$
(d) $\frac{x-1}{-1} = \frac{y+1}{2} = \frac{z-1}{1}$
(e) $x + 2 = y - 2 = z$

74. If the points $(1, 0, 0)$, $(0, 3, 0)$ and $(0, 0, 2)$ lie on a plane, then the unit normal vector n to the plane is

(a) $\frac{1}{\sqrt{14}}(\hat{i} + 3\hat{j} + 2\hat{k})$ (b) $\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$
(c) $\frac{1}{\sqrt{14}}(2\hat{i} + 3\hat{j} + \hat{k})$ (d) $\frac{1}{7}(3\hat{i} + 2\hat{j} + 6\hat{k})$
(e) $\frac{1}{7}(6\hat{i} + 2\hat{j} + 3\hat{k})$

75. The equation of the plane through the point $(1, -5, 3)$ and having a normal vector $\mathbf{n} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ is

(a) $2x + 2y + z = 9$ (b) $2x - 2y - z = 11$
(c) $2x + 2y - z = 9$ (d) $2x - 2y - z = 9$
(e) $2x - 2y - z = 13$

76. If θ is angle between the lines $\frac{x}{1} = \frac{y+1}{2} = \frac{z-1}{3}$ and $\frac{x+1}{3} = \frac{y}{2} = \frac{z}{1}$, then $\cos\theta =$

(a) 5/9 (b) 5/8 (c) 5/6 (d) 5/7
(e) 6/7

77. The distance from the point $(2, 2, 2)$ to the plane $2x - y + 3z = 5$ is equal to

(a) $\frac{3\sqrt{7}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{3\sqrt{14}}{7}$ (d) $\frac{3\sqrt{14}}{14}$
(e) $\frac{\sqrt{3}}{3}$

78. The angle between the planes $x = \sqrt{3}$ and $z = \sqrt{2}$ is equal to

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$
(e) 0

79. Three fair dice are rolled simultaneously. Let a, b, c be the numbers on the top of the dice. Then, the probability that $\min(a, b, c) = 6$ is

(a) $\frac{1}{216}$ (b) $\frac{1}{36}$ (c) $\frac{1}{6}$ (d) $\frac{11}{216}$
(e) $\frac{5}{6}$

80. If A and B are two events such that $P(A) = 0.5, P(B) = 0.4$ and $P(A \cap B) = 0.2$, then $P(A | (A \cup B))$ is equal to

(a) 6/7 (b) 5/6
(c) 5/7 (d) 4/7
(e) 1/2

81. There are 37 men and 33 women at a party. If a prize is given to one person chosen at random, then the probability that the prize goes to a woman is

(a) $\frac{33}{70}$ (b) $\frac{32}{70}$ (c) $\frac{33}{80}$ (d) $\frac{37}{70}$
(e) $\frac{37}{80}$

82. A fair coin is tossed twice. Given that the first toss resulted in head, then the probability that the second toss also, would result in head is

(a) 1/8 (b) 1/4 (c) 3/8 (d) 1/2
(e) 5/8

83. The coefficient of variation (CV) and the mean of a distribution are respectively 75 and 44. Then, the standard deviation of the distribution is

(a) 30 (b) 31 (c) 32 (d) 33
(e) 35

84. There are 4 red, 3 blue and 3 yellow marbles in an urn. If three marbles are drawn simultaneously, then the probability that the number of yellow marbles will be less than 2 is equal to

(a) $\frac{97}{120}$ (b) $\frac{49}{60}$ (c) $\frac{47}{60}$ (d) $\frac{59}{60}$
(e) $\frac{39}{60}$

85. In a box there are four marbles and each of them is marked with distinct number from the set $\{1, 2, 5, 10\}$. If one marble is randomly selected four times with replacement and the number on it noted, then the probability that the sum of numbers equals 18 is

(a) $\frac{1}{64}$ (b) $\frac{3}{16}$ (c) $\frac{5}{32}$ (d) $\frac{3}{32}$
(e) $\frac{1}{32}$

86. $\lim_{t\to 0}\left(\frac{(2t-3)(t-2)}{t}-\frac{3(t+2)}{t}\right)$ is equal to

(a) 10 (b) −10 (c) −7 (d) 7
(e) 5

87. If $f(x)=\begin{cases} x^2\sin\left(\frac{\pi}{6}x\right) & \text{for } x\le -3 \\ x\cos\left(\frac{\pi}{3}x\right) & \text{for } x>-3\end{cases}$, then the value of $\lim_{x\to -3^+} f(x)$ is equal to

(a) 3 (b) −3 (c) 9 (d) −9
(e) 0

88. $\lim_{x\to 0}\frac{\log(1+x)+1-e^x}{4x^2-9x}$ is equal to

(a) $\frac{-1}{9}$ (b) $\frac{1}{9}$ (c) $\frac{-1}{18}$ (d) $\frac{1}{18}$
(e) 0

89. $\lim_{t\to 0}\frac{\sin(t^2)}{t\sin(5t)}$ is equal to

(a) 5 (b) 25 (c) $\frac{1}{25}$ (d) $\frac{1}{5}$
(e) 0

90. Let $f(x)=\begin{cases} 3x+6, & \text{if } x\ge c \\ x^2-3x-1, & \text{if } x<c\end{cases}$, where $x\in R$ and c is a constant. The value of c for which f is continuous of R are

(a) −7, 1 (b) 1, 3 (c) −1, 7 (d) −1, 6
(e) 2, − 3

91. If $\lim_{x\to -2}\frac{3x^2+ax-2}{x^2-x-6}$ is a finite number, then the value of a is equal to

(a) 2 (b) 3 (c) 4 (d) 5
(e) 6

92. If $x=\sqrt{10^{\cos^{-1}\theta}}$ and $y=\sqrt{10^{\sin^{-1}\theta}}$, then $\frac{dy}{dx}$ is equal to

(a) xy (b) x/y (c) y/x (d) $-x/y$
(e) $\frac{-y}{x}$

93. If $y=e^{3\log(2x+1)}$, then $dy/dx=$

(a) $6e^{3\log(2x+1)}$ (b) $6\frac{e^{3\log(2x+1)}}{2x+1}$
(c) $\frac{e^{3\log(2x+1)}}{2x+1}$ (d) $\frac{e^{3\log(2x+1)}}{3(2x+1)}$
(e) $(2x+1)e^{3\log(2x+1)}$

94. If $x\sin y + y\sin x = \pi$, then $\frac{dy}{dx}$ at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is equal to

(a) 1 (b) $\pi/2$ (c) -1 (d) $-\pi/2$
(e) 0

95. Let $f(x) = \begin{cases} \tan x, & \text{if } 0 \le x \le \frac{\pi}{4} \\ ax + b, & \text{if } \frac{\pi}{4} < x < \frac{\pi}{2} \end{cases}$. If $f(x)$ is differentiable at $x = \pi/4$, then the values of a and b are respectively.

(a) $2, \frac{2-\pi}{2}$ (b) $2, \frac{4-\pi}{4}$ (c) $1, \frac{-\pi}{4}$ (d) $2, \frac{-\pi}{4}$
(e) $2, 1-\pi$

96. $\frac{d}{dx}\left(\frac{1}{x}\frac{d^2}{dx^2}\left(\frac{1}{x^3}\right)\right) =$

(a) $-36x^{-7}$ (b) $36x^{-7}$ (c) $72x^{-6}$ (d) $72x^{-7}$
(e) $-72x^{-7}$

97. Air is blown into a spherical balloon. If its diameter d is increasing at the rate of 3 cm/min, then the rate at which the volume of the balloon is increasing when $d = 10$ cm, is

(a) 120π cm^3/min (b) 150π cm^3/min
(c) 100π cm^3/min (d) 180π cm^3/min
(e) 210π cm^3/min

98. The equation of tangent to the circle $(x-5)^2 + y^2 = 25$ at $(2, 4)$ is

(a) $3x - 4y + 10 = 0$ (b) $x + y = 6$
(c) $2x - y = 0$ (d) $3x - 2y + 2 = 0$
(e) $3x - 4y - 10 = 0$

99. If x and y are both non-negative and if $x + y = \pi$, then the maximum value of $5\sin x \sin y$ is equal to

(a) 1 (b) $\sqrt{5}$ (c) 5 (d) -5
(e) 0

100. The normal to the curve $y = \sqrt{x}$ at the point $(25, 5)$ intersects the Y-axis at

(a) (0, 245) (b) (0, 255) (c) (255, 0) (d) (245, 0)
(e) (0, 100)

101. The function $f(x) = x^5 e^{-x}$ is increasing in the interval

(a) $(5, \infty)$ (b) $(4, \infty)$ (c) $(-4, \infty)$ (d) $(-\infty, 5)$
(e) $(-5, \infty)$

102. If $x + 13y = 40$ is normal to the curve $y = 5x^2 + \alpha x + \beta$ at the point $(1, 3)$, then the value of $\alpha\beta$ is equal to

(a) 15 (b) -6 (c) 6 (d) 13
(e) -15

103. Let $f(x) = \cos x$ for $0 \le x \le \frac{\pi}{3}$. Then, the value of c which satisfies the conclusion of the Mean Value Theorem for the function f on $[0, \pi/3]$ is equal to

(a) $\sin^{-1}\left(\frac{3}{2\pi}\right)$ (b) $\sin^{-1}\left(\frac{1}{3\pi}\right)$
(c) $\sin^{-1}\left(\frac{\pi}{12}\right)$ (d) $\sin^{-1}\left(\frac{1}{6\pi}\right)$
(e) $\sin^{-1}\left(\frac{\pi}{4}\right)$

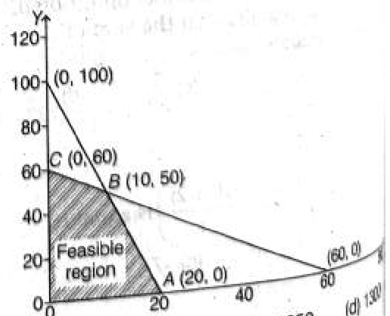
104. $\int \frac{e^{1/\sqrt{t}}}{t\sqrt{t}}\,dt =$

(a) $\frac{1}{2}e^{1/\sqrt{t}} + C$ (b) $\frac{-1}{2}e^{1/\sqrt{t}} + C$
(c) $2e^{e^{1/\sqrt{t}}} + C$ (d) $-2e^{1/\sqrt{t}} + C$
(e) $e^{e^{1/\sqrt{t}}} + C$

105. $\int \frac{\sin^{25} x}{\cos^{27} x}\,dx$ is equal to

(a) $\frac{\sin^{26}(x)}{26} + C$ (b) $\frac{\cos^{26}(x)}{26} + C$
(c) $\tan^{26}(x) + C$ (d) $\frac{\tan^{26}(x)}{26} + C$
(e) $26\tan^{26}(x) + C$

106. The feasible region for a LPP is shown in the figure below. Let $z = 50x + 15y$ be the objective function, then the maximum value of z is



(a) 900 (b) 1000 (c) 1250 (d) 1300
(e) 1520

107. $\int \frac{1}{x^3}\sqrt{1-\frac{1}{x^2}}\,dx =$

(a) $\frac{-1}{6}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$ (b) $\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

(c) $\frac{-1}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$ (d) $\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

(e) $\frac{-4}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

108. $\int (\tan^2(2x)-\cot^2(2x))\,dx =$

(a) $\frac{-1}{2}(\tan 2x+\cot 2x)+C$

(b) $2(\tan 2x+\cot 2x)+C$

(c) $\frac{1}{2}(\tan 2x-\cot 2x)+C$

(d) $\frac{-1}{2}(\tan 2x-\cot 2x)+C$

(e) $\frac{1}{2}(\tan 2x+\cot 2x)+C$

109. $\int \sin^3 x\,dx+\int \cos^2 x\sin x\,dx =$

(a) $-\cos x+C$ (b) $-\sin x+C$
(c) $x-\cos x+C$ (d) $x-\sin x+C$
(e) $\cos x-\sin x+C$

110. $\int \frac{dx}{x^2-x}=$

(a) $\log\frac{|x|}{|x-1|}+C$ (b) $\frac{-1}{x^2}+\log|x-1|+C$

(c) $x\log|x-1|+C$ (d) $\log\frac{|x-1|}{|x|}+C$

(e) $-x\log|x-1|+C$

111. The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\cot x}{\sin x}\,dx$ is equal to

(a) $\frac{-1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{-3}{2}$ (d) $\frac{3}{2}$
(e) 1

112. The area bounded by the curve $y=x(2-x)$ and the line $y=x$ is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{5}{6}$
(e) $\frac{2}{3}$

113. The value of $\int_{-1}^{2}(x-2|x|)\,dx$ is equal to

(a) $\frac{-1}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{-5}{2}$ (d) $\frac{-7}{2}$
(e) $\frac{-9}{2}$

114. The value of $\int_{-10}^{10}\frac{x^{10}\sin x}{\sqrt{1+x^{10}}}\,dx$ is equal to

(a) $\frac{1}{100}$ (b) $\frac{-1}{100}$ (c) $\frac{1}{50}$ (d) $\frac{-1}{50}$
(e) 0

115. If $f(x)=\begin{cases}\cos x & \text{for } x\ge 0\\ 2x & \text{for } x<0\end{cases}$, then the value of $\int_{-2}^{\frac{\pi}{2}} f(x)\,dx$ is equal to

(a) 2 (b) –2 (c) –3 (d) 3
(e) 0

116. The value of $\int_{0}^{\pi/16}\cos 6x\cos 2x\,dx$ is equal to

(a) $\frac{1+\sqrt{2}}{16}$ (b) $\frac{1+\sqrt{2}}{8}$

(c) $\frac{2+\sqrt{2}}{16}$ (d) $\frac{-1+\sqrt{2}}{16}$

(e) $\frac{-1+\sqrt{2}}{8}$

117. A particular solution of the differential equation $\frac{dy}{dx}=xy^2$ with $y(0)=1$ is

(a) $y=\frac{2-x^2}{2}$ (b) $y=\frac{2}{2-x^2}$

(c) $y=\frac{2}{x^2}-2$ (d) $y=\frac{x^2-2}{2}$

(e) $y=\frac{2}{x^2-2}$

118. The general solution of the differential equation $(x^2y^2+y)\,dx-(x-2x^3y)\,dy=0$ is

(a) $x^2y^2-\frac{y}{x}=C$ (b) $x^3y+\frac{x}{y}=C$

(c) $xy^2+\frac{y}{x}=C$ (d) $xy^2-\frac{y}{x}=C$

(e) $x^2y+\frac{y}{x}=C$

119. The integrating factor of the differential equation $4xdy - e^{-2y}dy + dx = 0$ is

(a) e^{-2y}
(b) e^{2x^2}
(c) e^{4y}
(d) e^{-4y}
(e) x^4

120. Consider the linear programming problem

Maximise $Z = 10x + 5y$

Subject to the constraints

$$2x + 3y \leq 120$$
$$2x + y \leq 60$$
$$x, y \geq 0$$

Then, the coordinates of the corner points of the feasible region are

(a) (0, 0), (30, 0), (0, 40) and (15, 30)
(b) (0, 0), (60, 0), (0, 40) and (15, 30)
(c) (0, 0), (30, 0), (0, 60) and (15, 30)
(d) (0, 0), (30, 0), (0, 40) and (30, 40)
(e) (0, 0), (60, 0), (0, 40) and (30, 40)

Answers

Physics & Chemistry

1.	(a)	2.	(c)	3.	(a)	4.	(c)	5.	(e)	6.	(b)	7.	(a)	8.	(e)	9.	(a)	10.	(c)
11.	(b)	12.	(b)	13.	(e)	14.	(c)	15.	(d)	16.	(b)	17.	(e)	18.	(e)	19.	(e)	20.	(b)
21.	(b)	22.	(a)	23.	(a)	24.	(c)	25.	(a)	26.	(e)	27.	(e)	28.	(b, c)	29.	(e)	30.	(c)
31.	(c)	32.	(a)	33.	(d)	34.	(d)	35.	(a)	36.	(d)	37.	(c)	38.	(c)	39.	(e)	40.	(b)
41.	(a)	42.	(c)	43.	(e)	44.	(d)	45.	(a)	46.	(e)	47.	(d)	48.	(d)	49.	(a)	50.	(c)
51.	(b)	52.	(d)	53.	(d)	54.	(d)	55.	(e)	56.	(b)	57.	(d)	58.	(e)	59.	(d)	60.	(e)
61.	(b)	62.	(d)	63.	(c)	64.	(b)	65.	(a)	66.	(c)	67.	(d)	68.	(e)	69.	(d)	70.	(b)
71.	(e)	72.	(b)	73.	(a)	74.	(d)	75.	(b)	76.	(a)	77.	(d)	78.	(c)	79.	(b)	80.	(c)
81.	(e)	82.	(b)	83.	(e)	84.	(b)	85.	(d)	86.	(e)	87.	(a)	88.	(c)	89.	(e)	90.	(d)
91.	(b)	92.	(b)	93.	(d)	94.	(c)	95.	(d)	96.	(c)	97.	(e)	98.	(a)	99.	(e)	100.	(b)
101.	(e)	102.	(a)	103.	(d)	104.	(b)	105.	(b)	106.	(c)	107.	(b)	108.	(c)	109.	(c)	110.	(a)
111.	(a)	112.	(a)	113.	(b)	114.	(d)	115.	(e)	116.	(a)	117.	(c)	118.	(b)	119.	(c)	120.	(a)

Mathematics

1.	(a)	2.	(e)	3.	(a)	4.	(b)	5.	(e)	6.	(e)	7.	(c)	8.	(b)	9.	(c)	10.	(c)
11.	(b)	12.	(a)	13.	(b)	14.	(e)	15.	(b)	16.	(d)	17.	(d)	18.	(d)	19.	(b)	20.	(c)
21.	(a)	22.	(e)	23.	(b)	24.	(c)	25.	(a)	26.	(c)	27.	(a)	28.	(c)	29.	(b)	30.	(b)
31.	(d)	32.	(e)	33.	(a)	34.	(b)	35.	(e)	36.	(c)	37.	(c)	38.	(e)	39.	(a)	40.	(d)
41.	(b)	42.	(e)	43.	(b)	44.	(a)	45.	(c)	46.	(e)	47.	(c)	48.	(e)	49.	(a)	50.	(b)
51.	(d)	52.	(c)	53.	(c)	54.	(a)	55.	(e)	56.	(a)	57.	(d)	58.	(b)	59.	(c)	60.	(a)
61.	(d)	62.	(e)	63.	(d)	64.	(c)	65.	(e)	66.	(e)	67.	(e)	68.	(c)	69.	(c)	70.	(b)
71.	(a)	72.	(b)	73.	(b)	74.	(e)	75.	(d)	76.	(d)	77.	(d)	78.	(d)	79.	(a)	80.	(c)
81.	(a)	82.	(d)	83.	(d)	84.	(b)	85.	(d)	86.	(b)	87.	(a)	88.	(e)	89.	(d)	90.	(c)
91.	(d)	92.	(e)	93.	(b)	94.	(c)	95.	(a)	96.	(e)	97.	(b)	98.	(a)	99.	(c)	100.	(b)
101.	(d)	102.	(e)	103.	(a)	104.	(d)	105.	(d)	106.	(c)	107.	(b)	108.	(e)	109.	(a)	110.	(d)
111.	(e)	112.	(d)	113.	(d)	114.	(e)	115.	(c)	116.	(a)	117.	(b)	118.	(d)	119.	(c)	120.	(a)

(*) None of the option is correct.

Answer *with* Explanations

Physics

1. *(a)* Power of lens, $P = \frac{1}{\text{Focal length (in m)}}$

Dimensional formula of power of lens is given as

$$[P] = \frac{1}{[L]} = [L^{-1}]$$
$$= [L^{-1}M^0T^0]$$

2. *(c)* The technology related with the Bernoulli's principle is used in aeroplane. The shape of wings of aeroplane is such that the air passes at a higher speed over the upper surface than the lower surface.

The region where velocity of air is more, creates low pressure and the region where velocity of air is less, creates high pressure. This pressure difference lifts the aeroplane in air.

3. *(a)* Sum of the given number

$$= 523.32 + 1.21524 + 107.3$$
$$= 631.83524$$

Since, the correct significant figures in given numbers are four in 107.3 with one decimal place. So, the final result should also have four significant figures upto one decimal place.

Hence, final result is rounded off to as,

$$\text{Sum} = 631.8$$

4. *(c)* Given, initial speed of cyclist,

$u = 0$, distance covered, $s = 120$ m

time, $t = 10$ s

By using equation of motion,

$$s = ut + \frac{1}{2}at^2$$
$$\Rightarrow \quad 120 = 0 \times 10 + \frac{1}{2} \times a \times 10^2 \Rightarrow 120 = 50a$$
$$\Rightarrow \quad a = \frac{120}{50} = 2.4 \text{ ms}^{-2}$$

5. *(e)* Given, position vector,

$$\mathbf{r} = 3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$$
$$\Rightarrow \quad |\mathbf{r}| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$$
$$\therefore \quad \hat{\mathbf{r}} = \frac{\mathbf{r}}{|\mathbf{r}|} = \frac{3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}}{3\sqrt{2}}$$
$$\hat{\mathbf{r}} = \frac{1}{\sqrt{2}}\hat{\mathbf{i}} + \frac{1}{\sqrt{2}}\hat{\mathbf{j}} \qquad \text{...(i)}$$

If α and β are angles made by position vector $\mathbf{r}$ with X-axis and Y-axis respectively, then

$\hat{\mathbf{r}} = (\cos\alpha)\,\hat{\mathbf{i}} + (\cos\beta)\,\hat{\mathbf{j}}$

Comparing with Eq. (i), we get

$$\therefore \quad \cos\alpha = \frac{1}{\sqrt{2}} \Rightarrow \cos\alpha = \cos 45^\circ$$
$$\Rightarrow \quad \alpha = 45^\circ$$

6. *(b)* In projectile motion, horizontal component of velocity remains constant during entire projectile motion, because no acceleration acts in horizontal direction. Only vertical component of velocity changes due to variation of acceleration due to gravity acting in downward direction.

7. *(a)* Velocity v of time is function of time t,

$$\Rightarrow \quad v = kt^2$$

where, k is constant.

i.e. $\quad v \propto t^2$

Hence, graph v *versus* t is parabola as shown in Fig (a).

8. *(e)* Gravitational force is not a contact force i.e., it is a non-contact force. It is an attractive field force field which does not include contact between the objects. Frictional force, buoyant force, air resistance and viscous force are example of contact force because these are attractive forces which require contact between the objects.

9. *(a)* Given, $m_1 = m$, $m_2 = 4m$

and $\quad K_1 : K_2 = 1 : 2$

We know that, kinetic energy K and momentum are related as

$$K = \frac{p^2}{2m}$$
$$\Rightarrow \quad p^2 = 2mK$$
$$\text{or} \quad p = \sqrt{2mK}$$
$$\therefore \quad \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2} \times \frac{K_1}{K_2}}$$
$$= \sqrt{\frac{m}{4m} \times \frac{1}{2}}$$
$$= \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}} \text{ or } 1 : 2\sqrt{2}$$

10. *(c)* According to Newton's second law of motion,

$$F = ma$$

where, F is force, m is mass and a is acceleration.

Observing the system in a tiny interval, where it travels an infinitesimal distance ds, i.e.,

$$F \cdot ds = ma\, ds$$

$$\Rightarrow \quad dW = m \cdot \frac{dv}{dt} \cdot ds \quad (\because W = Fs \text{ and } a = dv/dt)$$

$$\Rightarrow \quad dW = mvdv \quad (\because ds/dt = v)$$

Integrating both sides using appropriate limits,

$$\int_0^W dW = m\int_{v_i}^{v_f} vdv$$

$$\Rightarrow \quad W = m[v^2/2]_{v_i}^{v_f}$$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \text{KE}_f - \text{KE}_i$$

$$\Rightarrow \quad W = \Delta\text{KE}$$

Thus, work-energy theorem is an integral form of Newton's second law.

11. *(b)* In inelastic collisions, momentum is conserved but kinetic energy is not conserved, because some part of the energy is wasted in form of heat energy due to friction.

In elastic collision, both momentum and kinetic energy are conserved.

12. *(b)* Since, in tug of war, two opposite teams pull the rope with an equal and opposite force of 20 kN at each end of rope, then according to third law of motion at equilibrium condition,

Tension = Applied force

= 20 kN

13. *(e)* Given, mass of toy car, $m = 80 \text{ g} = 0.08 \text{ kg}$

Radius of circular path, $r = 0.8$ m

Centripetal force, $F = 10$ N

Since, $F = mv^2/r$

$$\Rightarrow \quad 10 = \frac{0.08\, v^2}{0.8} \Rightarrow v^2 = 100$$

$$\Rightarrow \quad v = 10 \text{ m/s}$$

14. *(c)* Given, mass of man, $M_m = 70$ kg

Mass of cart, $m_c = 30$ kg

Speed of cart, $v_c = 3 \text{ ms}^{-1}$

Relative speed of man w.r.t. cart, $v_{mc} = 4 \text{ ms}^{-1}$

$\Rightarrow$ Speed of man, $v_m = v_{mc} - v_c = 4 - 3 = 1 \text{ ms}^{-1}$

$\therefore$ Speed of centre of mass of system,

$$v_{\text{COM}} = \frac{M_m v_m + m_c v_c}{M_m + m_c}$$

$$= \frac{70 \times (-1) + 30 \times 3}{70 + 30} \quad (\because v_m \text{ is in opposite direction})$$

$$= \frac{20}{100} = 0.2 \text{ ms}^{-1}$$

15. *(d)* The moment of inertia of the circular platform is maximum when person is at the circumference and decreases as he moves towards centre with uniform velocity. Since, according to conservation of angular momentum, $I\omega$ = constant.

Thus, when moment of inertia (I) decreases, then angular velocity (ω) increases.

16. *(b)* Given, mass of solid metal cylinder (m_A)

= mass of hollow metal cylinder (m_B)

i.e. $\quad m_A = m_B$

and $\quad r_A / r_B = 2/1$

Moment of inertia of solid cylinder A,

$$I_{\text{solid}} = \frac{m_A r_A^2}{2}$$

Moment of inertia of hollow cylinder B,

$$I_{\text{hollow}} = m_B r_B^2$$

$$\therefore \quad \frac{I_{\text{solid}}}{I_{\text{hollow}}} = \frac{m_A r_A^2/2}{m_B r_B^2}$$

$$= \frac{m_A}{m_B} \cdot \left(\frac{r_A}{r_B}\right)^2 \cdot \frac{1}{2} = \frac{1}{2}\left(\frac{r_A}{r_B}\right)^2$$

$$= \frac{1}{2}\left(\frac{2}{1}\right)^2 = \frac{2}{1} \quad \text{or} \quad 2:1$$

17. *(e)* The angular momentum of a particle with respect to the origin will not be zero (or maximum), if the angle between the position vector and linear momentum is 90°.

18. *(e)* Given, mass of object, $m = 1$ kg

The minimum speed at which an object thrown from the surface of moon, will not come back on its surface, is called escape speed.

$\therefore$ Escape speed on the surface of moon $= \sqrt{2GM/R}$

where, $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

$$m = 7.34 \times 10^{22} \text{ kg}$$

$$R = 1.74 \times 10^6 \text{ m}$$

$\because$ Escape velocity $= \sqrt{2GM/R}$

$$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.34 \times 10^{22}}{1.74 \times 10^6}}$$

$$= 2.3 \text{ km/s}$$

19. *(e)* When the body is falling freely above the surface of earth, then it is acted upon by the force of gravity. But, the object is not applying its force to any surface i.e., reaction force on the body is zero during free fall. Therefore, the weight of the body is zero during free fall.

20. *(b)* Given, for satellites A and B,

$$R_A = 2R, R_B = R$$
$$v_A = 2v \text{ and } v_B = ?$$

We know that, speed of satellite,

$$v = \sqrt{\frac{GM}{R}}$$

$$\Rightarrow \quad \frac{v_B}{v_A} = \sqrt{\frac{R_A}{R_B}} = \sqrt{\frac{2R}{R}} = \sqrt{2}$$

$$\therefore \quad v_B = \sqrt{2}v_A = \sqrt{2} \times 2v$$
$$= 2\sqrt{2}v$$

21. *(b)* Given, mass, $m_1 = m_2 = 1$ kg

Distance, $r = 1$ cm $= 0.01$ m

$\therefore$ Gravitational potential energy,

$$U = \frac{Gm_1m_2}{r}$$
$$= \frac{G \times 1 \times 1}{0.01} = 100G \text{ joule}$$

22. *(a)* The relative viscosity of blood remains constant between 0°C and 37°C.

23. *(a)* Given, Young's modulus,

$$Y = 10 \times \text{Stress}$$

$$\Rightarrow \quad \frac{\text{Stress}}{\text{Strain}} = 10 \times \text{Stress} \Rightarrow \text{Strain} = \frac{1}{10} = 0.1$$

$$\Rightarrow \quad \Delta l / l = 0.1 \Rightarrow \Delta l = 0.1\, l$$

24. *(c)* Working of hydraulic lift is based on the principle of Pascal's law. According to this principle, an increase in pressure at any point inside a liquid at rest is transmitted equally and undiminished in all directions.

25. *(a)* Efficiency of Carnot engine is given as

$$\eta = 1 - \frac{T_2}{T_1} \quad \dots \text{(i)}$$

where, T_2 = temperature of sink

and T_1 = temperature of source.

From Eq. (i),

$$\frac{T_2}{T_1} = 1 - \eta$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{1}{1-\eta}$$

26. *(e)* In isochoric process, the volume remains constant. i.e. $\Delta V = 0$

According to first law of thermodynamics,

$$\Delta Q = \Delta W + \Delta U \quad \dots \text{(i)}$$

where, $\Delta W = p \cdot \Delta V = p \times 0 = 0$

$\therefore$ From Eq. (i),

$$\Delta Q = \Delta U$$

i.e. Heat exchanged = Change in internal energy

27. *(e)* Surface tension of a fluid is equal to force per unit length or change in energy per unit area.

Hence, statement given is option (e) is incorrect.

28. *(b, c)* If specific heat capacities of three identical silver cups A, B and C are s_A, s_B and s_C, then

$$s_A : s_B : s_C = 1 : 2 : 4$$

i.e. $\quad s_C > s_B > s_A$

which indicates that amount of heat energy released in cooling is maximum for the silver cup C and minimum for that of A.

So, cooling times of A, B and C can be arranged as

$$t_A < t_B < t_C$$

Hence, A cools faster than B and C. Also, B cools faster than C but slower than A.

29. *(e)* According to given situation, total degree of freedom,

$f = 3$ (translational) + 3 (rotational) + 2 (vibrational)

$$= 3 \times 1 + 3 \times 1 + 2 \times 2 \times 2 = 10$$

$$\therefore \quad \frac{C_p}{C_V} = 1 + \frac{2}{f} = 1 + \frac{2}{10}$$
$$= 1 + \frac{1}{5} = \frac{6}{5}$$

30. *(c)* $(v_{rms})_1 = 200$ m/s

$$T_1 = 27 + 273 = 300\text{K}$$
$$T_2 = 327 + 273 = 600\text{K}$$

We know that,

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow \quad v_{rms} \propto \sqrt{T}$$

$$\Rightarrow \quad \frac{(v_{rms})_2}{(v_{rms})_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{600}{300}} = \sqrt{2}$$

$$\Rightarrow \quad (v_{rms})_2 = \sqrt{2} \times (v_{rms})_1$$
$$= \sqrt{2} \times 200$$
$$= 200 \times 1.414 = 282.8 \text{ m/s}$$

31. **(c)** According to Clausius statement for the second law of thermodynamics, no process is possible in which heat transfer is taken from colder object to hotter object without doing work.

32. **(a)** Mean free path, $\bar{\lambda} = \frac{m}{\sqrt{2}\pi n r^2}$

$$\therefore \quad \bar{\lambda} \propto \frac{1}{nr^2}$$

Hence, mean free path of the molecule is inversely proportional to nr^2.

33. **(d)** Given, beat frequency = 4 Hz

$l_1 = 26$ cm and $l_2 = 25.2$ cm

If f be the frequency of fork, then as per question,

$$f - f_1 = 4 \quad \ldots(i)$$

and $$f_2 - f = 4 \quad \ldots(ii)$$

Also, $$\frac{f_1}{f_2} = \frac{l_2}{l_1} = \frac{25.2}{26} = \frac{63}{65}$$

$$\Rightarrow \quad f_1 = \frac{63}{65} \cdot f_2 \quad \ldots(iii)$$

Adding Eqs. (i) and (ii), we get

$$f_2 - f_1 = 8$$

Using value of f_1 from Eq. (iii) in above equation, we get

$$f_2 - \frac{63}{65} f_2 = 8 \Rightarrow f_2\left(1 - \frac{63}{65}\right) = 8$$

$$\Rightarrow \quad f_2 = \frac{8 \times 65}{2} = 260 \text{ Hz}$$

Using this value in Eq. (ii), we get

$$260 - f = 4$$

$$\Rightarrow \quad f = 260 - 4 = 256 \text{ Hz}$$

34. **(d)** Time period of simple pendulum,

$$T_1 = 3\text{s}, L_1 = L$$

When length of the pendulum is increased 4 times,

i.e. $\quad L_2 = 4L$, then $T_2 = ?$

We know that, time period of simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad T_2/T_1 = \sqrt{L_2/L_1} \Rightarrow T_2 = T_1\sqrt{4L/L}$$

$$= 2T_1 = 2 \times 3 = 6\text{s}$$

35. **(a)** The separation between two successive nodes is $\lambda/2$, where λ is wavelength.

36. **(d)** Given, displacement equation of particle executing SHM.

$$x = a \sin 2\pi ft$$

$$= a \sin \omega t \quad [\because \omega = 2\pi f]$$

Maximum acceleration,

$$\alpha_{max} = \omega^2 a$$

$$= (2\pi f)^2 a = 4\pi^2 f^2 a$$

Since, acceleration is maximum at highest displacement (amplitude).

At highest displacement, velocity of particle is zero.

i.e. $\quad v = 0$

$$\therefore \quad \frac{\alpha_{max}}{v} = \frac{4\pi^2 f^2 a}{0} = \infty$$

37. **(c)** Electric field intensity,

$$E' = 3E$$

Charge on proton, $q_p = e$

$\therefore$ Force experienced by a proton moving in an electric field,

$$F = q_p \cdot E'$$

$$= e \cdot 3E = 3eE$$

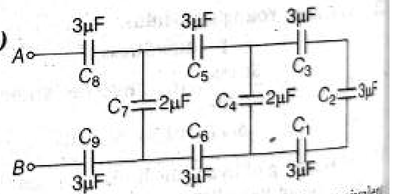
38. **(c)** We know that, work done by conservative force is zero, if initial and final points coincide. Hence, when a charge of -5μC is taken once round of a circle, then work done by electric force (conservative force) will be zero.

39. **(e)** Charge on doubly ionised helium atom (He^{++})

$$= 2e$$

$$= 2 \times 1.6 \times 10^{-19} \text{C} = 3.2 \times 10^{-19} \text{C}$$

40. **(b)**



C_1, C_2, C_3 are in series, hence their equivalent capacitance is given as

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{1}{1}$$

$$\Rightarrow C_{123} = 1\,\mu\text{F}$$

Again, C_{123} and C_4 are in parallel, hence their equivalent capacitance,

$C_{1234} = C_{123} + C_4 = 1 + 2 = 3\,\mu$F

Similarly, C_{1234}, C_5 and C_6 are in series,

$$C_{123456} = 1\mu\text{F}$$

$\Rightarrow C_{1234567} = 2 + 1 = 3\,\mu$F and $C_{123456789} = C_{AB} = 1\,\mu$F

41. **(a)** Charge on proton, $q_p = e$

$$= \text{Charge on electron}$$

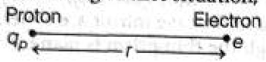
Charge on positron,

$$q_p' = \text{Charge on proton}$$

$$= q_p = e$$

$$\therefore \quad q_p = q_p' = e$$

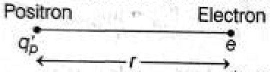
According to first situation,



$$F_1 = k \cdot \frac{q_p \cdot e}{r^2} = k\frac{e \cdot e}{r^2}$$

$$F_1 = ke^2/r^2 \quad \ldots \text{(i)}$$

According to second situation,



$$\therefore \quad F_2 = \frac{k \cdot q_p' e}{r^2} = \frac{k\, e \cdot e}{r^2} = \frac{ke^2}{r^2}$$

$$= F_1 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad F_1 = F_2$$

i.e. $F_1 : F_2 = 1:1$

42. ***(c)*** Kirchhoff's junction rule or Kirchhoff's current rule (KCL) is based on the law of conservation of charges whereas Kirchhoff's loop rule or Kirchhoff's voltage rule (KVL) is based on the law of conservation of energy.

43. ***(e)*** Resistivity of insulator and semiconductor both decrease with increase of temperature i.e. resistivity of insulator depends on temperature.

44. ***(d)*** From given colour coding, the value of resistance is

$$R = 20 \times 10^3 \Omega \pm 20\%$$

($\because$ Red $\rightarrow$ 2, Black $\rightarrow$ 0, Orange $\rightarrow$ 3 and no colour $\rightarrow \pm$ 20%)

$\therefore$ Valueof tolerance of resistor $= \pm 20000 \times \frac{20}{100}$

$$= \pm 4000\, \Omega$$

45. ***(a)*** Manganin is widely used to make wire bound standard resistors because it is an alloy having low temperature coefficient. Manganin is typically 85% copper, 13% manganese and 2% nickel.

46. ***(e)*** When a charged particle (either positive or negative) enters into a magnetic field perpendicular to it, then it moves on circular path into the magnetic field. Thus, when an electron and a proton moving with same velocity v enter into a uniform perpendicular magnetic field, then both move in circular paths.

47. ***(d)*** The current sensitivity of a moving coil galvanometer is given by

$$I_s = \frac{\theta}{I} = \frac{nBA}{C}$$

where, n is number of turns of coil, B is magnetic field, A is area of coil and C is restoring torque per unit twist.

Similarly, the voltage sensitivity of moving coil galvanometer,

$$V_s = \frac{\theta}{V} = \frac{nBA}{CR}$$

where, R is the resistance of coil.

So, when number of turns of the coil is doubled, the current sensitivity will be doubled as $I_s \propto n$, while the voltage sensitivity remains unchanged as $V_s \propto \frac{n}{R}$ and the resistance of coil increases in proportion to the number of turns of coil.

48. ***(d)*** Given, earth's magnetic field,

$$B = 0.4 \times 10^{-5}\,\text{T}$$

Radius of circular loop,

$$r = \pi\,\text{cm} = \pi \times 10^{-2}\,\text{m}$$

We know that, magnetic field at the centre of current carrying circular loop,

$$B = \frac{\mu_0 I}{2r}$$

$$\Rightarrow \quad 0.4 \times 10^{-5} = \frac{4\pi \times 10^{-7} \times I}{2 \times \pi \times 10^{-2}}$$

$$\Rightarrow \quad I = 0.2\text{A}$$

49. ***(a)*** A solenoid is a long coil of circular loops of insulated copper wire. Magnetic field lines are produced around the solenoid when a current is allowed to flow through it. The magnetic field produced by it is similar to the magnetic field of a bar magnet. Thus, bar magnet and solenoid both have attractive and directive properties and the magnetic field at the axial point is same for both.

50. ***(c)*** We know that, $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$

where, I_{rms} is rms value of alternating current and I_0 is the peak value of alternating current. Thus, rms value of AC current is $1/\sqrt{2}$ times the peak value of AC current.

51. ***(b)*** Magnetic energy stored per unit volume in a solenoid is given as $E_1 = \frac{B^2}{2\mu_0}$

where, B is magnetic field inside the solenoid.

The energy stored in a capacitor per unit volume is given by

$$E_2 = \frac{1}{2}\varepsilon_0 E^2$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{B^2/2\mu_0}{\frac{1}{2}\varepsilon_0 E^2} = \frac{B^2}{E^2}\cdot\frac{1}{\mu_0\varepsilon_0}$$

$$= \frac{B^2c^2}{E^2} \qquad [\because c = 1/\sqrt{\mu_0\varepsilon_0}]$$

52. *(d)* Transformer is used to convert low AC voltage in high AC voltage and *vice-versa* without changing the frequency of AC. Transformer can not work for DC source, because due to constant flux, there will be no emf induced in primary or secondary and hence transformer will be burnout.

Induction furnace is an application of eddy current. When armature coil of AC generator is rotated in uniform magnetic field, then emf is induced across it. Thus, mechanical energy is converted into electrical energy. *L-C-R* series resonance circuit is used to tune radio signals. The torque capacity of magnetic brake depends on the magnitude of magnetic flux generated.

53. *(d)* Given, AC voltage,

$$V = 330\sin(100\pi t)$$

$$\therefore \quad V_0 = 330\,\text{V}$$

and $I = 1.5\cos(100\pi t)$

$\therefore \; I_0 = 1.5\,\text{A}$

$\therefore$ Capacitive reactance,

$$X_C = \frac{V_{rms}}{I_{rms}} = \frac{V_0/\sqrt{2}}{I_0/\sqrt{2}} = \frac{V_0}{I_0} = \frac{330}{1.5} = 220\,\text{V}$$

54. *(d)* Radiowaves are used in cellular phones to transmit voice communication. Typical frequency range for cellular phones is 900 MHz to 1800 MHz which comes in ultrahigh frequency range.

55. *(e)* Some of the infrared radiation coming from sun passes through the atmosphere, but most it is absorbed and re-emitted in all directions by greenhouse gas molecules and clouds. This effect warms the earth and hence, it is called green house effect. Thus, infrared rays cause green house effect.

56. *(b)* Power of corrective lens, $P = -4.0\,\text{D}$

$\therefore$ Focal length, $F = \frac{1}{P} = \frac{1}{-4} = -0.25\,\text{m} = -25\,\text{cm}$

Since, power (or focal length) is negative, hence given lens is concave lens with focal length −25cm.

57. *(d)* As the density of a medium increases, its refractive index increases. Thus, the refractive index of air decreases with its density.

58. *(e)* When a plane wave front is incident on a thin prism, thin convex lens and a concave mirror separately, the wavefront(s) emerging out from the convex lens and concave mirror are spherical while that through the thin prism is plane.

59. *(d)* In Young's double slit experiment, when whole apparatus is immersed in a liquid of refractive index μ, then fringe width is decreased by μ times.

i.e. $$\beta_{liquid} = \frac{\beta_{air}}{\mu} \Rightarrow \beta_{air} = \mu\beta_{liquid} = \mu\beta$$

60. *(e)* Since, the reflected ray of *A* is partially polarised and reflected ray of *B* is completely polarised, hence, ray *B* falls on the glass slab at polarising angle i_p. Thus, $i_p = 60°$

$\therefore$ According to Brewster's law, refractive index of glass,

$$\mu = \tan i_p = \tan 60° = \sqrt{3} = 1.732$$

61. *(b)* The ratio of momenta of a proton, a neutron and an electron are given as

$$p_{proton} : p_{neutron} : p_{electron} = 3:2:1$$

Since, de-Broglie wavelength,

$$\lambda = \frac{h}{p} \Rightarrow \lambda \propto \frac{1}{p}$$

$$\therefore \lambda_{proton} : \lambda_{neutron} : \lambda_{electron} = \frac{1}{3}:\frac{1}{2}:\frac{1}{1} = 2:3:6$$

62. *(d)* Cadmium is not a photosensitive to visible light because its work function is too much high from the energy of visible light.

63. *(c)* Mass of substance,

$$m = 5\text{g} = 5\times10^{-3}\,\text{kg}$$

$$c = 3\times10^8\,\text{m/s}$$

According to Einstein's mass energy equivalence,

$\therefore$ Energy, $E = mc^2$

$$= 5\times10^{-3}\times(3\times10^8)^2$$

$$= 4.5\times10^{14}\,\text{J}$$

64. *(b)* For lighter nuclei, a nucleon has only few compatriots to bind with, so average binding energy per nucleon is low.

65. *(a)* Total binding energy in H-atom,

$$E = -3.4 \text{eV}$$

If n be the energy level, then

$$E = -\frac{13.6}{n^2}$$

$$\Rightarrow \quad -3.4 = \frac{-13.6}{n^2} \Rightarrow n^2 = 4$$

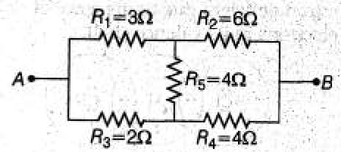
$$\Rightarrow \quad n = 2$$

$\therefore$ Angular momentum $= \frac{nh}{2\pi} = \frac{2 \times h}{2\pi} = \frac{h}{\pi}$

66. *(c)* In a nuclear reactor, the ratio of number of fission produced by a given generation of neutrons to the number of fission of the preceding generation is known as multiplication factor.

67. *(d)* LED stands for light emitting diode. It is a special purpose diode operated under forward bias. Zener diode and photo diode are operated in reverse bias.

68. *(e)* Since, potential at A is greater than the potential at B. So, both diodes given in the circuit diagram are in forward bias. Therefore, these can be represented as shortcircuit in the circuit diagram as drawn below

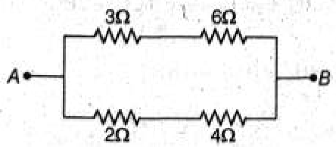


$$\frac{R_1}{R_2} = \frac{3}{6} = \frac{1}{2}$$

and $$\frac{R_3}{R_4} = \frac{2}{4} = \frac{1}{2}$$

Since, $$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Hence, this circuit is balanced Wheatstone bridge. Therefore, resistance R_5 is useless. Now, circuit diagram can be shown as



$\therefore$ Equivalent resistance,

$$R_{AB} = (3+6) \,||\, (2+4) = \frac{9 \times 6}{9+6} = 3.6\,\Omega$$

69. *(d)* When solar cell is illuminated with light photons of energy ($h\nu$) greater than the energy gap (E_g) of the semiconductor, then electron hole pair are generated due to absorption of photons. The three basic process involved are : generation, separation and collection of electron-hole pairs.

(i) Generation of electron-hole pair due to light ($h\nu > E_g$) close to the junction.

(ii) Separation of electrons and holes due to electric field of the depletion region. Electrons swept to n-side and holes to p-side.

(iii) The electrons reaching the n-side are collected by the front contact and holes reaching p-side are collected by the back contact. Thus, p-side becomes positive and n-side becomes negative giving rise to photo-voltage.

70. *(b)* In case (a), when $A = 0$ and $B = 1$

For OR gate, output,

$$Y = A + B = 0 + 1 = 1$$

For AND gate, output,

$$Y = AB = 0.1 = 0$$

For NAND gate, output, $Y = \overline{AB} = \overline{01} = \overline{0} = 1$

For NOR gate, output, $Y = \overline{A+B} = \overline{1+0} = \overline{1} = 0$

In case (b), when $A = 1$ and $B = 0$

For OR gate, output, $Y = A + B = 1 + 0 = 1$

For AND gate, output, $Y = AB = 1.0 = 0$

For NAND gate, output, $Y = \overline{AB} = \overline{1.0} = \overline{0} = 1$

For NOR gate, output, $Y = \overline{A+B} = \overline{1+0} = \overline{1} = 0$

Clearly, from case (a) and case (b), we see that OR and NAND gates give output of $Y = 1$.

71. *(e)* Given, carrier wave frequency,

$f = 200\,\text{MHz} = 2 \times 10^8$ Hz

$\therefore$ Wavelength of carrier wave,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5\,\text{m}$$

$\therefore$ Minimum length of dipole antenna,

$$l = \frac{\lambda}{2} = \frac{1.5}{2} = 0.75\text{m}$$

72. *(b)* In a communication system, transducer is used to convert energy from one form to another form.

Repeater is used to extend transmission, so that the signal can cover longer distance.

Amplifier is used to increase the strength of low frequency signal and attenuator is used to stop the flow of signal and antenna is used to radiate the radio signal.

Chemistry

73. *(a)* Among the given option one mole of aluminium sulphate contains the highest number of oxygen atoms, i.e. 12. The formula of aluminium sulphate is $Al_2(SO_4)_3$.

74. *(d)* Law of multiple proportions states that if two elements can combine to form more than one compound with each other the masses of one element that combine with a fixed mass of the other element are in ratios of small whole numbers.

Among the given options (a), (b), (c) and (e) illustrate the law of multiple proportions but H_2O and H_2S does not illustrate the law of multiple proportions.

75. *(b)* Number of electrons in $X^{2-} = 10$ electrons

$\therefore$ Number of electrons or protons in $X = 8\ (Z)$

And atomic mass = no. of protons + no. of neutrons

$\therefore$ Atomic mass of $X = 8 + 8 = 16$

Now, given that number of protons in $Y^{2+} = 12$

and number of neutrons in $Y = 1.5\times$ number of electron in atom X

$= 1.5 \times 8$

$\therefore$ Number of neutrons in $Y = 12$

Thus, atomic mass $= 12 + 12 = 24$

Then, the mass numbers of X and Y would be in the ratio of 16 : 24 or 2 : 3.

76. *(a)* Given, mass of particle $= 6.6 \times 10^{-31}$ kg

Velocity $= 1 \times 10^7 \text{ms}^{-1}$

Planck's constant $= 6.6 \times 10^{-31}$ Js

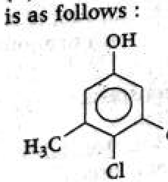
de-Broglie wavelength, $\lambda = ?$

$$\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}\ \text{Js}}{(6.6 \times 10^{-31} \times 1 \times 10^7)}\ \text{kg ms}^{-1}$$

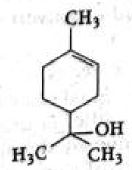
$= 1 \times 10^{-10}$ m

or $= 1$ Å $\qquad [1\text{ Å} = 10^{-10}\text{ m}]$

77. *(d)* The structure of chloroxylenol and terpineol is as follows :



Chloroxylenol



Terpineol

78. *(c)* Given, mass of the paricle $= 6.63 \times 10^{-28}$ g

$= 6.63 \times 10^{-31}$ kg

Particle located with an accuracy $= 1$ Å

$= 1 \times 10^{-10}$ m

Planck's constant, $h = 6.63 \times 10^{-34}$ Js

Uncertainty in velocity = ?

According to uncertainty principle,

$$\Delta p \cdot \Delta x \geq \frac{h}{4\pi}$$

$$m \cdot \Delta v \cdot \Delta x = \frac{h}{4\pi}$$

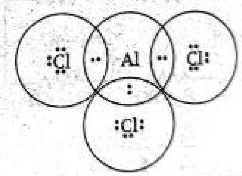
$$6.63 \times 10^{-31} \times \Delta v \times 1 \times 10^{-10} = \frac{6.63 \times 10^{-34}}{4 \times 3.14}$$

$$\therefore \Delta v = \frac{6.63 \times 10^{-34}}{6.63 \times 10^{-31} \times 3.14 \times 4 \times 10^{-10}}$$

$= 7.9 \times 10^5 \text{ms}^{-1}$

$\simeq 8 \times 10^5 \text{ms}^{-1}$

79. *(b)* Among the given option, $AlCl_3$ contains an incomplete octet as Al-atom in the molecule is electron deficient due to presence of only σ-electrons in its valence shell.



(Lewis dot structure of $AlCl_3$)

80. *(c)* Among the given option, only (c) involves th change from sp^2 to sp^3-hybridisation of the cen atom.

(c) $\underset{(sp^2)}{AlCl_3} + Cl^- \longrightarrow \underset{(sp^3)}{AlCl_4^-}$

(a) $\underset{(sp^3)}{CH_4} + 2Cl_2 \longrightarrow \underset{(sp^3)}{CH_2Cl_2} + 2HC$

(b) $\underset{(sp^3)}{NH_3} + H^+ \longrightarrow \underset{(sp^3)}{NH_4^+}$

(d) $\underset{(sp)}{H_2O} + H^+ \longrightarrow \underset{(sp^2)}{H_3O^+}$

(e) $\underset{(sp^2)}{PCl_3} + Cl_2 \longrightarrow \underset{(sp^3d)}{PCl_5}$

81. *(e)* The dipole - dipole interaction energy between rotating polar molecules is proportional to $\frac{1}{r^6}$.
where, 'r' is the distance between polar molecules.

82. *(b)* Metallic radius (r) of 'X' = 346.4 pm

The relation between edge-length (a) and radius of atom (r) for bcc lattice is $\sqrt{3}a = 4r$

$$\therefore \quad a = \frac{4r}{\sqrt{3}} = \frac{4 \times 346.4}{1.73} = 800 \text{ pm}$$

83. *(e)* Given,

Standard enthalpy of formation

$\Delta_f H, CH_4(g) = -75 \text{ kJ mol}^{-1}$

$\Delta_f H, H_2O(l) = -286 \text{kJ mol}^{-1}$,

$\Delta_f H, CO_2(g) = -393 \text{kJ mol}^{-1}$

$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

We know that, $\Delta H_{\text{reaction}}$

$= \Sigma\Delta H_f$ (product) $- \Sigma\Delta H_f$ (reactant)

$$\Delta H_{\text{reaction}} = [\Delta H_f(CO_2) + 2\Delta H_f(H_2O)] - [(\Delta H_f(CH_4)]$$

$$= [-393 + 2(-286)] - [-75]$$
$$= [-393 - 572] + 75$$
$$= [-965 + 75]$$
$$= -890 \text{kJmol}^{-1}$$

$$\text{Moles of } CH_4 = \frac{\text{Amount given}}{\text{Molar mass}} = \frac{3.2}{16} = 0.2$$

Thus, amount of heat evolved when 0.2 mole of methane gas is burnt $= \frac{-890}{0.2} = 178$ kJ

84. *(b)* Among the given options the correct relation between C_p and C_V for one mole of an ideal gas is given as $C_p = C_V + R$.

85. *(d)* Reactions that go almost to completion have high equilibrium constants.

For a reaction, $aR \rightleftharpoons bP$

$$K_C = \frac{[P]^b}{[R]^a}$$

The lesser value of equilibrium constant [i.e. in case of option (d)], shows that reaction is favoured in backward direction, i.e. concentration of reactant is much larger than the concentration of product.

86. *(e)* $2A \rightleftharpoons B + C; K_1 = 16$ ...(i)

$2B + C \rightleftharpoons 2X ; K_2 = 25$ ...(ii)

Multiply Eqs. (i), (ii) with $\frac{1}{2}$ we get

$$A \rightleftharpoons \frac{1}{2}B + \frac{1}{2}C; K_1 = (16)^{1/2} \quad ...(iii)$$

$$B + \frac{1}{2}C \rightleftharpoons X ; K_2 = (25)^{1/2} \quad ...(iv)$$

On adding equations (iii) and (iv) we get

$$A + \frac{1}{2}B \rightleftharpoons X; K = (16)^{1/2} \times (25)^{1/2}$$

$$A + \frac{1}{2}B \rightleftharpoons X ; K = 20$$

87. *(a)* Let the oxidation state of Br in Br_3O_8 be x and we know that oxidation state of oxygen is -2

$$\therefore \quad 3x + (8 \times -2) = 0$$
$$3x - 16 = 0$$
$$x = +\frac{16}{3}$$

Thus, the oxidation state of Br in Br_3O_8 is $\frac{16}{3}$.

88. *(c)* Given, $E^\circ_{Fe^{3+}/Fe^{2+}} = 0.77$ V

for $\quad Fe^{3+} + e^- \longrightarrow Fe^{2+}$

$E^\circ_{Ag^+/(Ag)(s)} = 0.80$

for $\quad Ag^+ + e^- \longrightarrow Ag$

Redox reaction,

$$Ag(s) + Fe^{3+} \longrightarrow Fe^{2+} + Ag^+$$

$$E^\circ_{\text{cell}} = E^\circ_{\text{O.P Ag(s)/Ag}^+} + E^\circ_{\text{R.P(Fe}^{3+}/\text{Fe}^{2+})}$$
$$= 0.8 + 0.77 = -0.03$$

Thus, the reaction is feasible, if E°_{cell} is positive but if E°_{cell} is negative then the reaction is not feasible. Thus, among the given option reaction of option (c) is not possible.

89. *(e)* Let the mass of the solution = 100 g

Given, that 20% urea = 20 g of urea and 80 g of water

$\therefore$ Mass of urea by solvent $= \frac{20}{80} \times 100 = 25\%$

For options (a), (b), (c) and (d) mass of urea dissolve in solvent is come out to be 25% but in (e) it come out to be 50%. Thus, option (e) does not conform to the required composition.

90. *(d)* Vapour pressure of pure liquid X,
$$p_x = 200 \text{ mm of Hg}$$
Vapour pressure of pure liquid Y,
$$p_y = 300 \text{ mm of Hg}$$
$$X_x = \frac{3}{5} \text{ and } X_y = \frac{2}{5}$$
$$\text{Now, } p_T = 200 \times \frac{3}{5} + 300 \times \frac{2}{5}$$
$$= 120 + 120 = 240 \text{ mm of Hg}$$

91. *(b)* Given, reaction $3A \longrightarrow$ product

The rate of decrease in $[A]$
$$= \frac{-\Delta A}{\Delta t} = -\left[\frac{0.1 - 0.4}{20}\right]$$
$$= \frac{-(-0.3)}{20} = \frac{0.3}{20} = 0.015$$

92. *(b)* Half-life period of a first order reaction
$$t_{1/2} = 20 \text{ min}$$
For first order reaction, $t_{1/2} = \dfrac{0.693}{k}$
$$k = \frac{0.693}{t_{1/2}} \quad \ldots(\text{i})$$
$$\text{Also } k = \frac{2.303}{t} \log\left[\frac{\text{Initial concentration}}{\text{Final concentration}}\right] \quad \ldots(\text{ii})$$
$\therefore$ For Eqs. (i) and (ii) we get
$$\frac{0.693}{t_{1/2}} = \frac{2.303}{t} \log\left[\frac{100}{100 - 99.9}\right]$$
$$t = \frac{2.303 \times 20}{0.693} \log\left[\frac{100}{0.1}\right]$$
$$= 199.39 \approx 200 \text{ min}$$

93. *(d)* The gas that is adsorbed to the maximum extent on 1g of activated charcoal at a given temperature is n-butane as its critical temperature is high among the given option.

We know, that higher is the critical temperature higher is the extent of adsorption.

94. *(c)* No appreciable activation energy is needed with regard to physisorption.

95. *(d)* The correct match is

A $\rightarrow$ (iii); B $\rightarrow$ (iv) ; C $\rightarrow$ (ii) ; D $\rightarrow$ (v); E $\rightarrow$ (i)

(A) Saline hydride	(iii) BeH_2
(B) Electron - deficient hydride	(iv) B_2H_6
(C) Electron precise hydride	(ii) CH_4
(D) Electron - rich hydride	(v) H_2O
(E) Metallic hydride	(i) CrH

96. *(c)* The metal which dissolves in liquid ammonia to give a blue - black solution due to formation of solvated electron is calcium. The ammoniated electrons absorb energy corresponding to a red region of visible light.

97. *(e)* Thermal decomposition of ammonium nitrate does not produce dinitrogen.

(a) $\underset{\text{Ammonium dichromate}}{(NH_4)_2Cr_2O_7} \xrightarrow{\Delta} \underset{\text{Nitrogen gas}}{N_2} + 4H_2O + \underset{\text{Chromium (III) oxide}}{Cr_2O_3}$

(b) $\underset{\text{Barium azide}}{Ba(N_3)_2} \xrightarrow{\Delta} Ba + 3N_2\uparrow$

(c) $\underset{\text{Ammonium chloride}}{NH_4Cl}\,(aq) + \underset{\text{Sodium nitrite}}{NaNO_2(aq)} \longrightarrow \underset{\text{Nitrogen gas}}{N_2(g)} + 2H_2O(l) + \underset{\text{Sodium chloride}}{NaCl(aq)}$

(d) $\underset{\text{Sodium azide}}{NaN_3} \longrightarrow \underset{\text{Sodium}}{Na(s)} + \frac{3}{2}\underset{\text{Nitrogen gas}}{N_2(g)}$

(e) $\underset{\text{Ammonium nitrate}}{NH_4NO_3(s)} \xrightarrow{\Delta} N_2O\uparrow + 2H_2O\uparrow$

98. *(a)* Dichlorodifluoromethane (CCl_2F_2) is also known as freon-12. It is used as refrigerant.

99. *(e)* Among the given options, Zn, Cd and Hg have high volatility as they have low boiling points.

100. *(b)* Cr^{2+} $(3d^4)$ acts as a reducing agent because it gets oxidised to Cr^{3+} $(3d^3)$ which gives half-filled t_{2g} level.

$$Cr^{2+} \longrightarrow Cr^{3+} + e^-$$

1	1	1	1	

$3d^4$

1	1	1		

Half-filled t_{2g} level

Whereas, Mn^{3+} $(3d^4)$ acts as an oxidising agent and is reduced to Mn^{2+} $(3d^5)$. It is a stable half-filled electronic configuration.

$$Mn^{3+} + e^- \longrightarrow Mn^{2+}$$

1	1	1	1	

$3d^4$

1	1	1	1	1

$3d^5$ (Stable configuration)

101. *(e)* $[Co(NH_3)_5NO_2]Cl_2$ and $[Co(NH_3)_5ONO]Cl_2$ are linkage isomers.

Linkage isomers are those isomers which have same molecular formula but differ in the linkage of ligand atom to the central metal atom.

102. *(a)* Magnesite is not an ore of iron. Its formula is $MgCO_3$.

(b) Haematite-Fe_2O_3 (c) Magnetite- Fe_3O_4
(d) Siderite-$FeCO_3$ (e) Iron pyrites- FeS_2

103. *(d)* The overall complex dissociation equilibrium constant for $[Cr(H_2O)_6]^{3+}$ ion $= 5\times10^{-12}$.

The overall stability constant of the complex

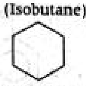
$$= \frac{1}{5\times10^{-12}} = 2\times10^{11}.$$

104. *(b)* The correct match is

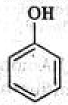
A → (iii) ; B → (v); C → (i) ; D → (ii) ; E → (iv)

(A) Alkane (iii) $CH_3—CH(CH_3)—CH_3$

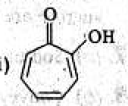
(Isobutane)

(B) Alicyclic compound (v) 

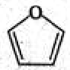
(Cyclohexane)

(C) Benzenoid aromatic compound (i) 

Phenol

(D) Non-benzenoid aromatic compound (ii) 

Tropolone

(E) Heterocyclic compound (iii) 

Furan

105. *(b)* Given,

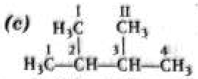
Percentage of carbon = 38.71 %

Percentage of hydrogen = 9.67 %

Percentage of oxygen = [100 − (38.71 + 9.67)]% = 51.62 %

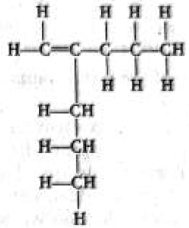
Element	Given amount (in%)	Molar mass in g/mol	Moles	Ratio of moles
C	38.71	12	$\frac{38.71}{12} = 3.23$	$\frac{3.23}{3.23} = 1$
H	9.67	1	$\frac{9.67}{1} = 9.67$	$\frac{9.67}{3.23} = 3$
O	51.62	16	$\frac{51.62}{16} = 3.23$	$\frac{3.23}{3.23} = 1$

So, the empirical formula of the given compound is CH_3O.

106. *(c)* 

2, 3-dimethylbutane

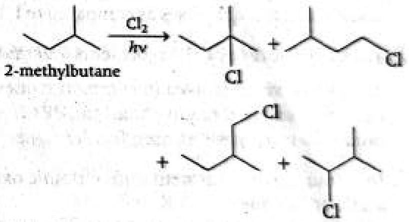
C_1, C_I, C_{II}, C_4 are primary carbons while C_2 and C_3 are tertiary carbons.

107. *(b)* 

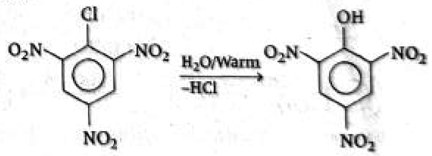
2, *n*-propylpent-1-ene

From the above structure it is clear that 2-*n*-propylpent-1-ene contains 23 σ-bonds and 1π-bond.

108. *(c)* Among the given options, 2-methylbutane gives four isomeric monochlorides on photochemical chlorination.



109. *(c)*



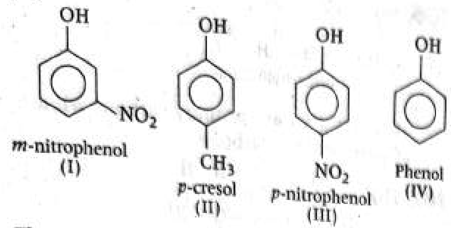
2,4,6-trinitrochlorobenzene

Due to the presence of 3 —NO_2 (electron withdrawing groups) 2, 4, 6-trinitrochlorobenzene is more reactive towards nucleophilic substitution reaction as compared to other aryl halides given in options as they require high temperature and pressure conditions to form corresponding phenols.

110. *(a)* 

Resorcinol
[Benzene-1,3-diol]

111. *(a)*



m-nitrophenol (I)   *p*-cresol (II)   *p*-nitrophenol (III)   Phenol (IV)

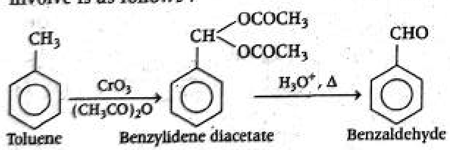
The acidity order of the given compounds is III > I > IV > II.

- The nitro-group is an electron-withdrawing group. The presence of this group in the *meta* or *para*-position decrease the electron density in the OH bond making proton donation easy. Now, *para*-position has more electron withdrawing effect than that of *meta*-position (—NO_2 showing only −*I*-effect). Therefore, III is more acidic than I and other given compounds.
- Phenol does not have any group. Thus is acidic character is less than I and III but more than II.
- Next, —CH_3 is an electron-donating group (+*I*) which increases the electron density in the —OH bond making O—H bond stronger. Due to which its acidic character decreases as compared to I, II and IV.

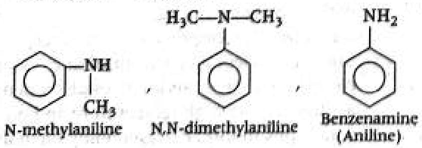
112. *(a)* $CH_3CH_2CH_2CH_2CHO$ represents valeraldehyde.

The prefix 'valer' is used in common names for five carbon atoms in straight chain. In IUPAC nomenclature, 'pent' is used for the same.

113. *(b)* Toluene on treatment with chromic oxide in acetic anhydride at 273 K to 283 K gives benzylidene diacetate. The reaction involve is as follows :

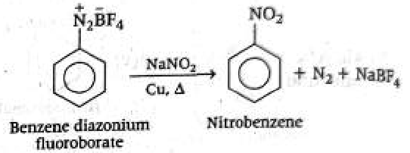


114. *(d)* CH_3NH_2 (Methanamine), $CH_3CH_2NH_2$ (Ethanamine),



Among the given compounds of amine benzamine is the weakest base as an electron withdrawing group is directly attached to it. Lone pair of N is delocalised on the benzene ring due to resonance and are not available for donation. N-methylaniline and N, N-dimethylaniline are stronger bases than aniline due to +*I*-effect of —CH_3. Whereas, ethanamine is the strongest base because an electron donating group (i.e. CH_2CH_3) (+*I* of —CH_3 < + *I* of —C_2H_5) is attached to it.

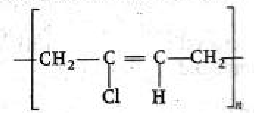
115. *(e)* The product formed when benzene diazonium fluoroborate is heated with aqueous sodium nitrite solution in the presence of copper is nitrobenzene. The reaction involved is as follows :



116. *(a)* Among the given options glycogen is a multibranched polysaccharide of glucose. Glucose-monosaccharide. Lactose, maltose and sucrose are disaccharides.

117. *(c)* Sodium iodide.

118. *(b)* Conveyor belt is manufactured from neoprene. Its structure is as follows :



Neoprene
[Synthetic rubber]

119. *(c)* Paracetamol is a non-narcotic analgesic drug.

120. *(a)* Nitrogen dioxide is the primary precursor of photochemical smog that can be metabolised by plants. CO_2 and SO_2 are not a part of photochemical smog whereas ozone and PAN are secondary precursors of it.

Mathematics

1. **(a)** Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3, 4\}$

Given that $R : A \longrightarrow B$ such that

$(a, b) \in R$ iff $a + b$ is even

Thus, $(1, 1), (1, 3), (2, 2), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4), (5, 1), (5, 3)$.

Relations exist and all $\{(1, 3), (2, 2), (1, 1), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4), (5, 1), (5, 3)\} \in R$

Thus, $n(R)$ equals to 10.

2. **(e)** The given function is $f(x) = (x^2 - 2x - 63)^{3/2}$, $x \in R$

We observe that $f(x)$ is defined for all x satisfying, $(x^2 - 2x - 63) \geq 0$

$$\Rightarrow \quad (x^2 - 9x + 7x - 63) \geq 0$$
$$\Rightarrow \quad x(x - 9) + 7(x - 9) \geq 0$$
$$\Rightarrow \quad (x + 7)(x - 9) \geq 0$$
$$\Rightarrow \quad -7 \leq x \leq 9$$
$$\Rightarrow \quad x \in (-\infty, -7] \cup [9, \infty)$$

3. **(a)** Let $A = \{x \in z; -1 \leq x < 4\}$

$$= \{-1, 0, 1, 2, 3\}$$

and $B = \left\{x \in z; 0 < \frac{x}{2} \leq 3\right\} = \{1, 2, 3, 4, 5, 6\}$

Thus, $A \cap B = \{1, 2, 3\}$

4. **(b)** The given function is

$$f(x) = \begin{cases} x + 2 \ , & x < 1 \\ 4x - 1 \ , & 1 \leq x \leq 3 \\ x^2 + 5 \ , & x > 3 \end{cases}$$

Then, check continuity at $x = 1$ and 3 and all other points f is continuous. So, option A, D, E are false.

Now, check continuity at $x = 1$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x + 2) = 3$$

and $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (4x - 1) = 3$ and $f(1) = 3$

Thus, $\lim_{x \to 1^+} f(x) = f \lim_{x \to 1^-} (x) = f(1)$

Now, check continuity at $x = 3$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (4x - 1) = 12 - 1 = 11$$

and $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x^2 + 5) = 9 + 5 = 14$

$$\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$$

Hence, $f(x)$ is not continuous at $x = 3$.

5. **(e)** Given that $\odot$ be a binary operation on $Q - \{0\}$ such that $a \odot b = \frac{a}{b}$

Then, $1 \odot (2 \odot (3 \odot 4))$

$$= 1 \odot \left(2 \odot \left(\frac{3}{4}\right)\right) \qquad \left[\text{Using } a \odot b = \frac{a}{b}\right]$$
$$= 1 \odot \left(\frac{2}{3/4}\right)$$
$$= 1 \odot \left(\frac{8}{3}\right)$$
$$= \frac{1}{8/3} = \frac{3}{8}$$

6. **(e)** Let $f : R \to R$ defined by $f(x) = \cos x$

Clearly, $f(0) = \cos 0 = 1$ and $f(2\pi) = \cos 2\pi = 1$

So different element in R may have same image. Hence, f is not an injective on one-one.

Again, since the value of $\cos x$ lie between -1 to 1 it follows that the range of $f(x)$ is not equal to R its codomain. So f is not onto.

But $f(-x) = \cos(-x) = \cos x = f(x)$

$\therefore f$ is even function [$\because$ if $f(-x) = f(x)$, then f is even function]

7. **(c)** Given that

$n(A \cup B) = 97$, $n(A \cap B) = 23$ and $n(A - B) = 39$

So, we know that

$$n(A - B) = n(A) - n(A \cap B)$$
$$\Rightarrow \quad 39 = n(A) - 23$$
$$\Rightarrow \quad n(A) = 62$$

Thus, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\Rightarrow \quad 97 = 62 + n(B) - 23$$
$$\Rightarrow \quad n(B) = 97 - 62 + 23$$
$$= 120 - 62 = 58$$

Thus, $n(B) = 58$

8 **(b)** Given that $z = \frac{8 + 4i}{1 + 3i}$

$$= \frac{(8 + 4i)(1 - 3i)}{(1 + 3i)(1 - 3i)}$$
$$= \frac{8 + 12 - 24i + 4i}{1 + 9}$$
$$= \frac{20 - 20i}{10}$$
$$\Rightarrow \quad z = 2 - 2i$$

Let α be the acute angle given by

$$\tan\alpha = \left|\frac{\text{Im}(z)}{\text{Re}(z)}\right| = \left|\frac{-2}{2}\right| = 1$$

$$\Rightarrow \quad \alpha = \tan^{-1}(1) = \frac{\pi}{4}$$

We observe that $\text{Re}(z) = 2 > 0$ and $\text{Im}(z) = -2 < 0$.
So that the point representing z lies in the 4th quadrant

$$\therefore \quad \arg(z) = -\alpha = \frac{-\pi}{4}$$

9. **(c)** We know that

$$|z_1| + |z_2| \geq |z_1 - z_2|$$

$$\Rightarrow \quad |z+1| + |z-2| \geq |(z+1) - (z-2)|$$

$$\Rightarrow \quad |z+1| + |z-2| \geq 3$$

Hence, minimum value of $|z+1| + |z-2|$ is 3.

10. **(c)** Given, $z = \dfrac{(3+i)(7-i)^2}{3-i}$

$$= \frac{(3+i)(49-14i-1)}{3-i}$$

$$= \frac{(3+i)(48-14i)}{3-i}$$

$$= \frac{144+14+48i-42i}{3-i} = \frac{158+6i}{3-i}$$

$$= \frac{(158+6i)(3+i)}{9+1}$$

$$= \frac{474-6+18i+158i}{10}$$

$$= \frac{468+176i}{10} = 46.8 + 17.6i$$

$$z = \sqrt{(46.8)^2 + (17.6)^2}$$

$$z = \sqrt{2190 + 310} = \sqrt{2500} = 50$$

11. **(b)** Given, $\left[\dfrac{5i}{(1-i)(2-i)(3-i)}\right]^{50}$

$$= \left[\frac{5i}{(2-3i-1))(3-i)}\right]^{50}$$

$$= \left[\frac{5i}{(1-3i))(3-i)}\right]^{50}$$

$$= \left[\frac{5i}{3-10i-3}\right]^{50} = \left[\frac{-1}{2}\right]^{50} = \left(\frac{1}{2}\right)^{50}$$

12. **(a)** Since, $z^4 = 7 - 5i$

$$\Rightarrow \quad (\bar{z})^4 = 7 + 5i$$

$$\text{Im}((\bar{z})^4) = \text{Im}(7+5i) = 5$$

13. **(b)** Given, $\left(\dfrac{1+i}{1-i}\right)^{75} - \left(\dfrac{1-i}{1+i}\right)^{75}$

$$\Rightarrow \quad \left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^{75} - \left(\frac{(1-i)(1-i)}{(1+i)(1-i)}\right)^{75}$$

$$= \left(\frac{2i}{2}\right)^{75} - \left(\frac{-2i}{2}\right)^{75} = (i)^{75} - (-i)^{75}$$

$$= (i)^{75} + (i)^{75} = 2(i)^{75} = -2i$$

Modulus of above expression is $|-2i| = 2$

14. **(e)** Given that, if z_1 and z_2 are two different complex number with $|z_2| = 1 \Rightarrow z_2\bar{z}_2 = 1$

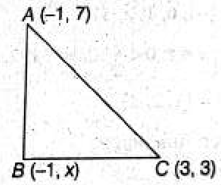
Thus, $\left|\dfrac{1-\bar{z}_1 z_2}{z_1 - z_2}\right|$

$$= \left|\frac{(1-\bar{z}_1 z_2)(1-z_1\bar{z}_2)}{(z_1-z_2)(\bar{z}_1-\bar{z}_2)}\right|$$

$$= \left|\frac{1-\bar{z}_1 z_2 - z_1\bar{z}_2 + \bar{z}_1 z_2 z_1 \bar{z}_2}{z_1\bar{z}_1 - \bar{z}_1 z_2 - z_1\bar{z}_2 + z_2\bar{z}_2}\right|$$

$$= \left|\frac{1-\bar{z}_1 z_2 - z_1\bar{z}_2 + z_1\bar{z}_1}{1-\bar{z}_1 z_2 - z_1\bar{z}_2 + z_1\bar{z}_1}\right| = 1$$

15. **(b)**



Since, ABC is isosceles triangle.

So, $AB = BC$

$$\Rightarrow \sqrt{(-1+1)^2 + (x-7)^2} = \sqrt{(-1-3)^2 + (x-3)^2}$$

$$\Rightarrow (x-7)^2 = 16 + (x-3)^2$$

$$\Rightarrow x^2 - 14x + 49 = 16 + x^2 - 6x + 9$$

$$\Rightarrow 24 = 8x$$

$$\Rightarrow x = 3$$

16. **(d)** The sum of first 24 terms of the series $9 + 13 + 17 + \ldots\ldots$ is

$$= \frac{24}{2}\{2\times 9 + (24-1)\cdot 4\}$$

$$= 12\{18 + 23\times 4\}$$

$$= 12\{18 + 92\} = 12\times 110 = 1320$$

17. **(d)** In the AP,

$$T_{16} = a + (16-1)d = 67 \quad \ldots \text{(i)}$$

and $$T_{17} = a + (17-1)d = 72 \quad \ldots \text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$a + 15d = 67$$
$$a + 16d = 72$$
$$\underline{(-)\quad(-)\quad(-)}$$
$$-d = -5$$

$\Rightarrow$ $d = 5$

Thus, $a = 67 - 15 \times 5 = 67 - 75 = -8$

Hence, first term is -8.

18. *(d)* Given, first term of GP series = 3

and nth term is $= (a_n) = ar^{n-1} = 3r^{n-1}$

Also, $a_2 + a_3 = 60$

$\Rightarrow 3r^{2-1} + 3r^{3-1} = 60$ [where r is an common ratio]

$$\Rightarrow r + r^2 = 20$$
$$\Rightarrow r^2 + r - 20 = 0$$
$$\Rightarrow (r+5)(r-4) = 0$$
$$\Rightarrow r = -5, 4$$

Thus, common ratio of GP is 4 or -5

19. *(b)* Given, nth term of the series $= n + (-1)^{n-1}$, $n = 1, 2, 3 \ldots\ldots$

$$\because \quad S_n = \frac{n}{2}\{2a + (n-1)d\}$$
$$\Rightarrow \quad S_{40} = \frac{40}{2}\{2 \times 1 + (40-1) \cdot 1\}$$

$[\because a = 1, a + d = 2 \text{ and } d = 2 - 1 = 1]$

$$= 20\{2 + (39) \cdot 1\} = 20 \times (41) = 820$$

20. *(c)* Since, 11th term of $\sum_{n=0}^{\infty} a_n r^{n-1}$ is ar^{n-1}

Thus, 11th term of geometric series

$$\sum_{n=0}^{20} 2 \times (-2)^n = (2) \times (-2)^{11-1}$$
$$= 2 \times (-2)^{10} = 2 \times 1024 = 2048$$

21. *(a)* Let S_n be the sum of the first n terms of the series $a_1 + a_2 + \ldots$ and given that $S_n = n^2 + 4n$

Sum of $(n-1)$ terms is $S_{n-1} = (n-1)^2 + 4(n-1)$

$$= n^2 - 2n + 1 + 4n - 4$$
$$= n^2 + 2n - 3$$

We know,

sum of n term is equal to sum of $(n-1)$ th terms + nth term

$$\Rightarrow \quad S_n = S_{n-1} + (n\text{th term})$$

$\Rightarrow$ (nth term) $= S_n - S_{n-1}$

$$= (n^2 + 4n) - (n^2 + 2n - 3)$$
$$= 2n + 3$$

22. *(e)* Given, $t_n = \frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2$, $n = 1, 2, 3 \ldots\ldots$

$$\Rightarrow \quad t_{10} = \frac{1}{10}\sum_{k=1}^{10}\left(\frac{k}{10}\right)^2$$
$$= \frac{1}{10}\left\{\left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^2 + \ldots + \left(\frac{10}{10}\right)^2\right\}$$
$$= \frac{1}{10}\left\{\frac{1^2}{100} + \frac{2^2}{100} + \ldots + \frac{10^2}{100}\right\}$$
$$= \frac{1}{10} \times \frac{1}{100}\{1^2 + 2^2 + \ldots + 10^2\}$$
$$= \frac{1}{1000}\left\{\frac{10 \cdot (10+1)(2 \times 10 + 1)}{6}\right\}$$
$$= \frac{1}{1000}\left\{\frac{110 \times 21}{6}\right\}$$
$$= \frac{231}{600} = \frac{77}{200}$$

23. *(b)* The number of arrangement containing all the seven letter of the word ALRIGHT that begains with LG is $5! = 120$ ways

24. *(c)* We can have either 4 digit or 5 digit number. First consider 4 digit number. We can take 6, 7, 8 at the thousand place. The next three places can be filled in $= {}^4C_3 \times 3$ way

So we get ${}^4C_3 \times 3 = \frac{4! \cdot 3}{(4-3)!}$

$$= 4 \times 3 \times 2 \times 3 = 12 \times 6 = 72$$

We get 72 such number. For 5 digit number, we can choose them in 5! ways i.e. 120 ways.

Thus, total number of numbers is equal to $72 + 120 = 192$.

25. *(a)* The number of digit = 9

Given set is {2, 4, 6, 8, 10, 12, 14, 16, 18}

Exactly 4 elements are to be chosen of the above set. So, number of ways ${}^9C_4$.

$$= \frac{9!}{4! \times 5!}$$
$$= \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 5!} = 126$$

26. *(c)* Given, ${}^{11}P_r = 7920$ and ${}^{11}C_r = 330$

$$\Rightarrow \quad \frac{11!}{(11-r)!} = 7920 \quad \ldots\text{(i)}$$
$$\Rightarrow \quad \frac{11!}{(11-r)!r!} = 330 \quad \ldots\text{(ii)}$$

From Eq. (ii), we get

$$\frac{11!}{(11-r)!} = 330\cdot r!$$

$\Rightarrow$ $7920 = 330\cdot r!$

$\Rightarrow$ $r! = 24 = 4!$

$\Rightarrow$ $r = 4$

Hence, the value of r is 4.

27. *(a)* Given that the binomial expansion

$(x-2y^2)^9 = (x+(-2y^2))^9$

$= {}^9C_0(x)^9\cdot(-2y^2)^0 + {}^9C_1(x)^8(-2y^2)$
$\quad + {}^9C_2(x)^7(-2y^2)^2 + {}^9C_3(x)^6(-2y^2)^3 + \dots$

Thus, the coefficient of x^6y^6 is

$${}^9C_3(-2)^3 = -\frac{9!}{3!\times 6!}\times 8$$

$= -672$

28. *(c)* $(3+x)^{10}$ we know that,

$= \{2+(1+x)\}^{10}$

$= {}^{10}C_0(2)^{10}\cdot(1+x)^0 + {}^{10}C_1(2)^9\cdot(1+x)^1$
$\quad + {}^{10}C_2(2)^8(1+x)^2 + \dots + {}^{10}C_{10}(2)(1+r)^{10}$

Thus, $a_0 + a_1 + \dots + a_{10}$

$= {}^{10}C_0(2)^{10} + {}^{10}C_1(2)^9 + \dots + {}^{10}C_{10}(2)^1$

$= (2+1)^{10} = 3^{10}$

29. *(b)* If ${}^nC_5 + {}^nC_6 = {}^{51}C_6$

We know that, n and r be non-negative integer such that $r \le n$. Then ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Here, $r = 6$ and $n+1 = 51$

$\therefore n = 50$

30. *(b)* Given, $A = \begin{bmatrix}3 & 4\\1 & -2\end{bmatrix}$ and $AB = \begin{bmatrix}-5 & 41\\5 & -13\end{bmatrix}$

$\because \det A = -6-4 = -10$

$$\Rightarrow \quad A^{-1} = \frac{\text{adj}A}{|A|} = \frac{\begin{bmatrix}-2 & -4\\-1 & 3\end{bmatrix}}{-10}$$

$$= \frac{1}{10}\begin{bmatrix}2 & 4\\1 & -3\end{bmatrix}$$

$$AB = \begin{bmatrix}-5 & 41\\5 & -13\end{bmatrix}$$

Multiply both sides by A^{-1}, we get

$$B = \frac{1}{10}\begin{pmatrix}2 & 4\\1 & -3\end{pmatrix}\begin{pmatrix}-5 & 41\\5 & -13\end{pmatrix} = \frac{1}{10}\begin{pmatrix}10 & 30\\-20 & 80\end{pmatrix}$$

$$B = \begin{pmatrix}1 & 3\\-2 & 8\end{pmatrix}$$

$$\Rightarrow \quad B^T = \begin{pmatrix}1 & -2\\3 & 8\end{pmatrix}$$

$\Rightarrow$ $|B^T| = 8+6 = 14$

31. *(d)* Given, $A = \begin{bmatrix}2 & 1 & -2\\1 & 1 & -1\\1 & 0 & 3\end{bmatrix}$

$\Rightarrow |A| = 2(3-0) - 1(+1+3) - 2(-1)$

$= 6-4+2 = 4$

$$\text{adj } A = \begin{bmatrix}3 & -4 & -1\\-3 & 8 & +1\\1 & 0 & 1\end{bmatrix}$$

Now, $B = |A|\,\text{adj}\,A$

$$B = 4\begin{bmatrix}3 & -4 & -1\\-3 & 8 & 1\\1 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \quad B = \begin{bmatrix}12 & -16 & -4\\-12 & 32 & 4\\4 & 0 & 4\end{bmatrix}$$

$\Rightarrow$ $|B| = 12(128) + 16(-48-16) - 4(-128) = 1024$

32. *(e)* Given, $\begin{vmatrix}x & 4 & 0\\2 & 2 & -x\\1 & 1 & 1\end{vmatrix} = 0$

$\Rightarrow$ $x(2+x) + 4(-x-2) + 0(2-2) = 0$

$\Rightarrow$ $x^2+2x-4x-8 = 0$

$\Rightarrow$ $x^2-2x-8 = 0$

$(x-4)(x+2) = 0$

$\Rightarrow$ $x = 4, -2$

33. *(a)* Given, $A = [2, 0, 6]$ $B = \begin{bmatrix}3 & 5\\7 & -2\\6 & 6\end{bmatrix}$

$$AB = [2 \;\; 0 \;\; 6]_{1\times 3}\begin{bmatrix}3 & 5\\7 & -2\\6 & 6\end{bmatrix}_{3\times 2}$$

$= [42, 46]$

34. *(b)* Given A is non-singular matrix and

$$A^{-1} = \frac{1}{2}\begin{bmatrix}-10 & -4\\2 & 1\end{bmatrix}$$

Since, $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

$\Rightarrow$ adj $A = (|A|)(A^{-1})$

Since, $AA^{-1} = I$

$\Rightarrow \quad |A| . |A^{-1}| = 1$

$\Rightarrow \quad |A| = \frac{1}{|A^{-1}|}$

Now, $|A^{-1}| = \frac{1}{4}(-10 + 8) = \frac{1}{4}(-2) = \frac{-1}{2}$

Thus, $|A| = \frac{-2}{1} = -2$

Now, adj $A = |A| \cdot A^{-1} = -2 \cdot \frac{1}{2}\begin{bmatrix} -10 & -4 \\ 2 & 1 \end{bmatrix}$

$= -\begin{bmatrix} -10 & -4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 4 \\ -2 & -1 \end{bmatrix}$

35. *(e)* Given $\begin{vmatrix} \sin\alpha & \cos(\alpha+\theta) & \cos\alpha \\ \sin\beta & \cos(\beta+\theta) & \cos\beta \\ \sin\gamma & \cos(\gamma+\theta) & \cos\gamma \end{vmatrix}$

$= \begin{vmatrix} \sin\alpha & \cos\alpha\cos\theta - \sin\alpha\sin\theta & \cos\alpha \\ \sin\beta & \cos\beta\cos\theta - \sin\beta\sin\theta & \cos\beta \\ \sin\gamma & \cos\gamma\cos\theta - \sin\gamma\sin\theta & \cos\gamma \end{vmatrix}$

$= \begin{vmatrix} \sin\alpha & 0 & \cos\alpha \\ \sin\beta & 0 & \cos\beta \\ \sin\gamma & 0 & \cos\gamma \end{vmatrix}$

$[\because C_2' \rightarrow C_2 - \{C_3\cos\theta - C_1\sin\theta\}]$

$= 0$

36. *(c)* Given, $-2 \le \frac{3x+2}{2} < 7$

$\frac{3x+2}{2} \ge -2$

$\Rightarrow \quad 3x + 2 \ge -4$

$\Rightarrow \quad 3x \ge -6$

$\Rightarrow \quad x \ge -2$

$\frac{3x+2}{2} < 7$

$\Rightarrow \quad 3x < 14 - 2$

$\Rightarrow \quad x < 4$

$\therefore \{x : -2 \le x < 4\}$ is required solution set.

37. *(c)* Given inequality, $|3x + 4| \le 7$

$\Rightarrow \quad (3x + 4) \le 7$ and $-(3x + 4) \le 7$

$\Rightarrow \quad 3x \le 3 \Rightarrow 3x + 4 \ge -7$

$\Rightarrow \quad x \le 1 \Rightarrow x \ge \frac{-11}{3}$

Thus, $x \in \left[\frac{-11}{3}, 1\right]$

38. *(e)* Given that, the solution set of the inequality $|a + 3x| \le 6$ is $\left[\frac{-8}{3}, \frac{4}{3}\right]$

$\Rightarrow \quad a + 3x \le 6$ and $-a - 3x \le 6$

$\Rightarrow \quad x \le \frac{6-a}{3}, \Rightarrow -x \le \frac{6+a}{3}$

$\Rightarrow \quad x \ge \frac{-6-a}{3}$

According to question, $\frac{6-a}{3} = \frac{4}{3}$

$\Rightarrow \quad a = 6 - 4 = 2$

39. *(a)* (i) For every positive real number x, $x - 10$ is a positive $\rightarrow$ false statement (take $x = 1$)

(ii) Let $n \in \pi e$ then n^2 even $\Rightarrow n$ even (True)

(iii) If $n \in \pi e$ is odd then n^2 is also odd (True)

40. *(d)* If $\cos\theta = \frac{5}{11}$

$\Rightarrow \sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{5}{11}\right)^2}$

$= \sqrt{1 - \frac{25}{121}} = \sqrt{\frac{96}{121}} = \pm\frac{4\sqrt{6}}{11}$

Since, $\tan\theta < 0$ and $\cos\theta > 0$ thus θ in 4th quadrant.
So, $\sin\theta < 0$.

Thus, $\sin\theta = -\frac{4\sqrt{6}}{11}$

41. *(b)* Given, $(\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2$

Since, α and β are two acute angle of right triangle.
Thus, $(\alpha + \beta) = 90°$

$\Rightarrow \quad \beta = 90° - \alpha$

$= \{\sin\alpha + \sin(90° - \alpha)\}^2 + \{\cos\alpha + \cos(90° - \alpha)\}^2$

$= \{\sin\alpha + \cos\alpha\}^2 + \{\cos\alpha + \sin\alpha\}^2$

$= 2(\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha) = 2(1 + \sin 2\alpha)$

42. *(e)* Since, we know that

$-1 \le \sin 3x \le 1, \forall x \in R$

$\Rightarrow \quad -2 \le 2\sin 3x \le 2, \forall x \in R$

$\Rightarrow \quad -2 + 1 \le 2\sin 3x + 1 \le 2 + 1, \forall x \in R$

$\Rightarrow \quad -1 \le 2\sin 3x + 1 \le 3, \forall x \in R$

$\Rightarrow \quad -1 \le f(x) \le 3, \forall x \in R$

$\Rightarrow \quad f(x) \in [-1, 3]$

43. *(b)* Given, $g(x) = 5\cot\left(\frac{\pi}{3}x + \frac{\pi}{6}\right) + 2$

Period of a function $g(x)$ is $\frac{\pi}{\left|\frac{\pi}{3}\right|} = \frac{\pi \times 3}{\pi} = 3$

44. (*a*) If $-\pi < \theta < 0$ and $\cos\theta = \frac{-12}{13}$

Thus, $1 - 2\sin^2\frac{\theta}{2} = \frac{-12}{13}$

$\Rightarrow \quad 2\sin^2\frac{\theta}{2} = \frac{25}{13}$

$\Rightarrow \quad \sin\frac{\theta}{2} = \pm\sqrt{\frac{25}{26}} = \pm\frac{5}{\sqrt{26}}$

$= \pm\frac{5\sqrt{26}}{26}$

Since $\quad -\pi < \theta < 0$

$\Rightarrow \quad -\frac{\pi}{2} < \frac{\theta}{2} < 0$

So in this region $\sin\theta < 0$

$\Rightarrow \quad \sin\frac{\theta}{2} = \frac{-5\sqrt{26}}{26}$

45. (*c*) Given, $\cos\theta = 2 - 3\sin\left(\frac{\theta}{2}\right)$

$\Rightarrow \quad 1 - 2\sin^2\frac{\theta}{2} = 2 - 3\sin\frac{\theta}{2}$

$\Rightarrow \quad 2\sin^2\frac{\theta}{2} - 3\sin\frac{\theta}{2} + 1 = 0$

$\Rightarrow \quad \left(2\sin\frac{\theta}{2} - 1\right)\left(\sin\frac{\theta}{2} - 1\right) = 0$

$\Rightarrow \quad 2\sin\frac{\theta}{2} = 1, \sin\theta/2 = 1$

$\Rightarrow \quad \sin\theta/2 = \frac{1}{2} = \sin\frac{\pi}{6}$

$\Rightarrow \quad \sin\frac{\theta}{2} = 1 = \sin\frac{\pi}{2}$

$\Rightarrow \quad \frac{\theta}{2} = \frac{\pi}{6}$

$\Rightarrow \quad \theta = \frac{\pi}{3}$

$\Rightarrow \quad \theta = \pi, \frac{\pi}{3}$

46. (*e*) Given, $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

$\because$ We know that $\cos^{-1}(\cos\theta) = 2\pi - \theta$ if $\pi \le \theta \le 2\pi$

$= 2\pi - \frac{7\pi}{6}$

$= \frac{12\pi - 7\pi}{6} = \frac{5\pi}{6}$

47. (*c*) Given, $\tan\left(\sin^{-1}\left(\frac{7}{25}\right)\right)$

$= \frac{\frac{7}{25}}{\sqrt{1 - \left(\frac{7}{25}\right)^2}} \quad \left[\because \tan(\sin^{-1} x) = \frac{x}{\sqrt{1 - x^2}}\right]$

$= \frac{\frac{7}{25}}{\frac{\sqrt{625 - 49}}{25}} = \frac{\frac{7}{25}}{\frac{\sqrt{576}}{25}} = \frac{7}{24}$

48. (*e*) Given, $\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{200}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{200}\right)\right)$

$= \cos\frac{\pi}{2} = 0 \quad \left[\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right]$

49. (*a*) Since, if a line parallel to another line, their slopes will be equal.

So, slope is -3

Thus equation of line whose slope -3 and passing through the point $(3, -2)$ is

$y + 2 = -3(x - 3)$

$\Rightarrow \quad y = -3x + 7$

50. (*b*) Equation formed by two intercepts is

$\frac{x}{a} + \frac{y}{b} = 1$

Given that, $a = b$

Since, the line passes through the point $(2, 3)$.

So, $\frac{2}{a} + \frac{3}{a} = 1$

$\Rightarrow \quad \frac{5}{a} = 1$

$\Rightarrow a = 5$ and $b = 5$

Thus, equation of the line $= x + y = 5$.

51. (*d*) Given that, the line $y = mx + c$ is perpendicular to $y = 1 + x$

Thus, slope of $y = mx + c$ is

$= -\frac{1}{\text{slope of } y - 1 + x}$

$\Rightarrow \quad m = -1$

$\therefore \quad y = -x + c$ also passes through $(1, 2)$

$\Rightarrow \quad 2 = -1 + c$

$\Rightarrow \quad c = 3$

52. *(c)* Let $A=(-1,2)$, $B(1,3)$ and $C(a,b)$

all these points are collinear and given that B divides AC such that $\frac{BC}{AB}=\frac{8}{1}$

Since, the point $B(1,3)$ divide the line segment $A(-1,2)$ and $C(a,b)$ in the ratio 1 : 8 internally.

Thus, $(1,3)=\left(\frac{a\cdot 1+(-8)}{8+1},\frac{b+2\cdot 8}{9}\right)$

$\Rightarrow \quad \frac{a-8}{9}=1, \frac{b+16}{9}=3$

$\Rightarrow \quad a=9+8=17$

$\Rightarrow \quad b=27-16=11$

Thus, coordinate of $C=(17,11)$

53. *(c)* Given that,

the three line are

$$2x-3y+5=0 \quad \dots(i)$$
$$9x-5y+14=0 \quad \dots(ii)$$
$$3x-7y+\lambda=0 \quad \dots(iii)$$

Since, given that these above three lines are concurrent if they passes through a common point. Thus, point of intersection of any two lines lies on the third line.

Now, find point of intersection from Eqs. (i) and (ii), we get

Multiply by $9\times$ Eq. (i) and $2\times$ Eq. (ii) and subtracting Eq. (i) from Eq. (ii)

$$18x-27y=-45$$
$$18x-10y=-28$$
$$(-) \qquad (+)$$
$$-17y=-17$$

$\Rightarrow \quad y=1$ and $x=-1$

Thus, $(-1,1)$ point lies on the line $3x-7y+\lambda=0$

Hence, $\quad 3\cdot(-1)-7\cdot 1+\lambda=0$

$\Rightarrow \quad -3-7+\lambda=0$

$\Rightarrow \quad \lambda=10$

Thus, the value of $\lambda=10$

54. *(a)* Given, equation of line $y=x+2 \quad \dots(i)$

and equation of circle $(x-2)^2+y^2=16 \quad \dots(ii)$

Put $y=x+2$ in Eq. (ii), we get

$$(x-2)^2+(x+2)^2=16$$

$\Rightarrow \quad x^2-4x+4+x^2+4x+4=16$

$\Rightarrow \quad 2x^2=8$

$\Rightarrow \quad x=\pm 2$

If $x=2$, then $y=4$

and if $x=-2$, then $y=0$

Then, the point of intersection are $(2,4)$ and $(-2,0)$.

55. *(e)* Let the equation of the circle is

$$x^2+y^2+2gx+2fy+c=0 \quad \dots(i)$$

It passes through $(0,0)$, $(3,1)$, and $(1,3)$.

Thus $(0,0)$ Put in Eq. (i), we get $c=0$

and $(3,1)$

Put in Eq. (i), we get $10+6g+2f$

$\Rightarrow \quad 3g+f=-5 \quad \dots(ii)$

and $(1,3)$ put in Eq. (ii), we get

$$10+g+3f=0$$

$\Rightarrow \quad g+3f=-5 \quad \dots(iii)$

From Eqs. (i) and (ii), we get

$$2g-2f=0\Rightarrow g=f$$

Thus, $g=\frac{-5}{4}$ and $f=-\frac{5}{4}$ and $c=0$

$\therefore$ From Eq. (i), we get

$$x^2+y^2-\frac{5}{2}x-\frac{5}{2}y=0$$

$\Rightarrow \quad 2x^2+2y^2-5x-5y=0$

56. *(a)* Equation of circle touching X-axis at $(5,0)$ means centre of a circle lie on $(5,k)$.

$\Rightarrow \quad (x-5)^2+(y-k)^2=k^2$

$\Rightarrow \quad x^2-10x+25+y^2-2ky=0 \quad \dots(i)$

Now, S touches the line $y=10$

So, perpendicular distance from the centre (y,k) to the given line is equal to radius of circle i.e.

$$k=\left|\frac{k-10}{\sqrt{1^2}}\right|$$

$\Rightarrow \quad k=-k+10$

$\Rightarrow \quad k=5$

Thus, equation of circle is

$$x^2+y^2-10x-10y+25=0$$

57. *(d)* Radius of circle $x^2+y^2+ax+by+3=0$ is

$$\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2-3}=2 \text{ (given)}$$

$\Rightarrow \quad \frac{a^2+b^2}{4}=4+3=7$

$\Rightarrow \quad a^2+b^2=28$

Now, (a,b) lies on the circle

$$x^2+y^2+ax+by+3=0$$

Then, $\quad x^2+y^2=28$

58. *(b)* Given equation of a line $2x-3y+c=0$ and given equation of the parabola is $x^2=-8y$. Thus, focus of this parabola is $(0,-2)$.

Thus, the given line passes through $(0, -2)$.

Thus, $\quad 2\cdot 0 - 3\cdot(2) + c = 0$

$\Rightarrow \quad c = -6$

Hence, the value of c is -6.

59. *(c)* Given, equation of the ellipse is

$$x^2 + 7y^2 - 14x + 28y + 49 = 0$$

$$\Rightarrow \quad (x^2 - 14x + 49) + 74y^2 + 28y = 0$$

$$\Rightarrow \quad (x-7)^2 + 7\{y^2 + 4y + 4\} - 28 = 0$$

$$\Rightarrow \quad (x-7)^2 + 7(y+2)^2 = 28$$

$$\Rightarrow \quad \frac{(x-7)^2}{28} + \frac{(y+2)^2}{4} = 0$$

Thus, centre is $(7, -2)$.

60. *(a)* The end points on major axis of an ellipse are $(2, 4)$ and $(2, -8)$.

Thus, $2a = \sqrt{(2-2)^2 + (4+8)^2} = 12$

$\Rightarrow \quad a = 6$

Distance between foci is 4

$\Rightarrow \quad 2ae = 4$

$$\Rightarrow \quad e = \frac{4}{2} = \frac{1}{3}$$

$$\Rightarrow \quad \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{3}$$

$$\Rightarrow \quad 1 - \frac{b^2}{36} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{8}{9} = \frac{b^2}{36}$$

$$\Rightarrow \quad b^2 = 32$$

Thus, equation of an ellipse is

$$\frac{(y+2)^2}{36} + \frac{(x-2)^2}{32} = 1$$

61. *(d)* Given that $(-1, 0)$ and $(3, 0)$ and foci of an ellipse.

So, distance between two foci is $\sqrt{(3+1)^2 + 0} = 4$

$\therefore \quad 2ae = 4$

$\Rightarrow \quad ae = 2$

Also given length of major axis $= 6$

$\Rightarrow \quad 2a = 6$

$\Rightarrow \quad a = 3$

$\therefore \quad ae = 2$

$$\Rightarrow \quad e = \frac{2}{3} \Rightarrow \sqrt{1 - \frac{b^2}{a^2}} = \frac{2}{3}$$

$$\Rightarrow \quad 1 - \frac{b^2}{a^2} = \frac{4}{9}$$

$$\Rightarrow \quad \frac{5}{9} = \frac{b^2}{9} \Rightarrow b = \sqrt{5} \quad [\because a = 3]$$

Thus, length of minor axis is $2\sqrt{5}$.

62. *(e)* Given equation of the hyperbola is

$$\frac{(x-3)^2}{9} - \frac{4(y-1)^2}{45} = 1$$

Thus $a^2 = 9$ and $b^2 = \frac{45}{4}$

$$\text{Eccentricity of hyperbola} = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{45/4}{9}}$$

$$= \sqrt{1 + \frac{5}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

63. *(d)* Given, $\mathbf{a} \times \mathbf{b} = 7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$

and $\mathbf{a}\cdot\mathbf{b} = -20$

We know,

$$\Rightarrow \quad |\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a}\cdot\mathbf{b})^2$$

$$\Rightarrow \quad (\sqrt{49 + 81 + 100})^2 = |\mathbf{a}|^2 \cdot |\mathbf{b}|^2 - (-20)^2$$

$$\Rightarrow \quad 230 + 400 = |\mathbf{a}|^2 \cdot |\mathbf{b}|^2$$

$$\Rightarrow \quad |\mathbf{a}|^2 \cdot |\mathbf{b}|^2 = 630$$

64. *(c)* Given, $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, $\mathbf{a}\cdot\mathbf{b} = 4$

and $\quad \mathbf{a} + \mathbf{b} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$

Clearly, $\quad \mathbf{b} = (4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}) - (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$

$\quad \mathbf{b} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - (\lambda + 3)\hat{\mathbf{k}}$

Now, $\mathbf{a}\cdot\mathbf{b} = 4$

$[\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}]\cdot[(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) + (\lambda + 3)\hat{\mathbf{k}}] = 4$

$\Rightarrow \quad 3 - 8 - 3(\lambda + 3) = 4$

$\Rightarrow \quad -5 - 3\lambda - 9 = 4$

$\Rightarrow \quad -3\lambda = 18$

$\Rightarrow \quad \lambda = -6$

65. *(e)* We have, $|\mathbf{a}| = \sqrt{14}$, $|\mathbf{b}| = \sqrt{10}$ and $|\mathbf{a} - \mathbf{b}| = \sqrt{24}$

We know that

$$|\mathbf{a} - \mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\mathbf{a}\cdot\mathbf{b}$$

$$\Rightarrow \quad 24 = 14 + 10 - 2|\mathbf{a}||\mathbf{b}|\cos\theta$$

$$\Rightarrow \quad 24 = 24 - 2\sqrt{14}\sqrt{10}\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{24 - 24}{2\sqrt{14}\sqrt{10}} = 0$$

66. *(e)* We have,

$|\mathbf{a}| = 10$ and $|\mathbf{b}| = 5$

$= (\mathbf{a} + 2\mathbf{b}) \cdot (\mathbf{a} - 2\mathbf{b})$

$= \mathbf{a} \cdot \mathbf{a} - \mathbf{a} \cdot 2\mathbf{b} + 2\mathbf{b}\mathbf{a} - 4\mathbf{b} \cdot \mathbf{b}$

$= |\mathbf{a}|^2 - 4|\mathbf{b}|^2$

$= (10)^2 - 4(5)^2 = 100 - 100 = 0$

67. *(e)* Given $\mathbf{a} = \hat{i} - 3\hat{j} + 3\hat{k}$ and $\mathbf{b} = 2\hat{i} + \hat{j} - 3\hat{k}$

Now, $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 3 \\ 2 & 1 & -3 \end{vmatrix}$

$= \hat{i}(9 - 3) - \hat{j}(-3 - 6) + \hat{k}(1 + 6)$

$= 6\hat{i} + 9\hat{j} + 7\hat{k}$

$\therefore \quad (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = (6\hat{i} + 9\hat{j} + 7\hat{k}) \cdot (2\hat{i} + \hat{j} - 3\hat{k})$

$= 12 + 9 - 21 = 0$

68. *(c)* Let $\mathbf{a} = \alpha\hat{i} + 3\hat{j} + 0\hat{k}$

and $\quad \mathbf{b} = \hat{i} + 0\hat{j} + 0\hat{k}$

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

$$\cos\frac{\pi}{4} = \frac{(\alpha\hat{i} + 3\hat{j} + 0\hat{k}) \cdot (\hat{i} + 0\hat{j} + 0\hat{k})}{\sqrt{\alpha^2 + 9 + 0}\,\sqrt{1}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{\alpha}{\sqrt{\alpha^2 + 9}}$$

$\Rightarrow \quad \alpha^2 + 9 = 2\alpha^2$

$\therefore \quad \alpha^2 = 9$

$\alpha = \pm 3$

69. *(c)* Given,

$\mathbf{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$\mathbf{b} = \hat{i} + 3\hat{j} + 2\hat{k}$

Unit vector along the direction of

$$\mathbf{a} + \mathbf{b} = \frac{\mathbf{a} + \mathbf{b}}{|\mathbf{a} + \mathbf{b}|}$$

Now, $\mathbf{a} + \mathbf{b} = (2\hat{i} + 3\hat{j} - 4\hat{k}) + (\hat{i} + 3\hat{j} + 2\hat{k})$

$= 3\hat{i} + 6\hat{j} - 2\hat{k}$

$|\mathbf{a} + \mathbf{b}| = \sqrt{(3)^2 + (6)^2 + (-2)^2} = \sqrt{49} = 7$

So, the required unit vector is

$$\frac{\mathbf{a} + \mathbf{b}}{|\mathbf{a} + \mathbf{b}|} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$$

$$= \frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$$

70. *(b)* We have, $|\mathbf{u}| = 3, |\mathbf{v}| = 2$ and $|\mathbf{u} \times \mathbf{v}| = 3$

Let θ be the angle between $\mathbf{u}$ and $\mathbf{v}$.

$\Rightarrow |\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| \cdot |\mathbf{v}| \sin\theta$

$\Rightarrow 3 = 3 \times 2 \sin\theta$

$\therefore \quad \sin\theta = \frac{1}{2}$

$$\theta = \frac{\pi}{6}, \left(\pi - \frac{\pi}{6}\right)$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

71. *(a)* Given, point is $(-1, -2, -3)$ its position vector is $\mathbf{a} = -1\hat{i} - 2\hat{j} - 3\hat{k}$ and perpendicular to the vector $= x\hat{i}$

Required equation of plane is $(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$

$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (x\hat{i}) = (-1\hat{i} - 2\hat{j} - 3\hat{k}) \cdot x\hat{i}$

$\Rightarrow \quad x^2 = -x$

$\Rightarrow \quad x = -1$

72. *(b)* The equation of the line L_1 is

$$\frac{x - 0}{1 - 0} = \frac{y - 0}{2 - 0} = \frac{z - 0}{3 - 0}$$

$\Rightarrow \quad x = \frac{y}{2} = \frac{z}{3} = \lambda \quad$ ... (i)

and the equation of line L_2 is

$$\frac{x - 2}{1} = \frac{y - 3}{1} = \frac{z - 4}{1}$$

$\Rightarrow \quad x - 2 = y - 3 = z - 4 = \mu \quad$ ... (ii)

Then, any point on line (i) is

$P(\lambda, 2\lambda, 3\lambda)$ and any point on line (ii) is

$Q(\mu + 2, \mu + 3, \mu + 4)$

Lines (i) and (ii) will intersect, if

$(\lambda, 2\lambda, 3\lambda) = (\mu + 2, \mu + 3, \mu + 4)$

For some particular value of λ and μ

$\Rightarrow \quad \lambda = \mu + 2, 2\lambda = \mu + 3, 3\lambda = \mu + 4$

Then, we get $\lambda = 1$ and $\mu = -1$

Hence, line (i) and (ii) intersect and their point of intersection is $P(1, 2 \times 1, 3 \times 1)$ [put $\lambda = 1$]

i.e. $P(1, 2, 3)$

73. *(b)* Given, a line passing through the point $(1, -1, 1)$ and parallel to the line joining the points $(-2, 2, 0)$ and $(-1, 1, 1)$.

The equation of parallel line

$$\frac{x + 2}{1} = \frac{y - 2}{-1} = \frac{z}{1}$$

$\because$ Direction ratios of the given lines are $(1, -1, 1)$.

Direction ratios of a line parallel to the given line are proportional to (1, −1, 1).

∴ The required line will be parallel to the vector.

$$\mathbf{b} = \hat{i} - \hat{j} + \hat{k}$$

Hence, the required equation of line is given by $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$

$$x\hat{i} + y\hat{j} + z\hat{k} = \hat{i} - \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-1}{1}$$

$$1 - x = 1 + y = 1 - z$$

74. *(e)* Equation of the plane, $\frac{x}{1} + \frac{y}{3} + \frac{z}{2} = 1$

$$\Rightarrow \frac{6x + 2y + 3z}{6} = 1$$

$$\Rightarrow 6x + 2y + 3z = 6$$

$$\mathbf{N} = 6\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\underline{n} = \frac{6\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{36 + 4 + 9}} = \frac{1}{7}(6\hat{i} + 2\hat{j} + 3\hat{k})$$

75. *(d)* Here, point is (1, − 5, 3), its position vector is $\mathbf{a} = \hat{i} - 5\hat{j} + 3\hat{k}$ and normal vector, **n** to the plane is $\mathbf{n} = 2\hat{i} - 2\hat{j} - \hat{k}$

Now, vector equation of the plane passing through one point is given by $(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$

$$\therefore [\mathbf{r} - (\hat{i} - 5\hat{j} + 3\hat{k}] \cdot 2\hat{i} - 2\hat{j} - \hat{k} = 0$$

$$\Rightarrow \mathbf{r} \cdot (2\hat{i} - 2\hat{j} - \hat{k}) = (\hat{i} - 5\hat{j} + 3\hat{k}) \cdot (2\hat{i} - 2\hat{j} - \hat{k})$$

$$\Rightarrow \mathbf{r} \cdot (2\hat{i} - 2\hat{j} - \hat{k}) = 2 + 10 - 3$$

$$\Rightarrow \mathbf{r} \cdot (2\hat{i} - 2\hat{j} - \hat{k}) = 9$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} - 2\hat{j} - \hat{k}) = 9$$

[on putting $\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$]

$$\Rightarrow 2x - 2y - z = 9$$

76. *(d)* Given equation of lines are $\frac{x}{1} = \frac{y+1}{2} = \frac{z-1}{3}$

and $\frac{x+1}{3} = \frac{y}{2} = \frac{z}{1}$

Here, direction ratios of two lines are (1, 2, 3) and (3, 2, 1).

Let θ be the acute angle between the given lines, then.

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{|1 \times 3 + 2 \times 2 + 3 \times 1|}{\sqrt{(1)^2 + (2)^2 + (3)^2}\sqrt{(3)^2 + (2)^2 + (1)^2}}$$

$$\cos\theta = \frac{10}{\sqrt{14} \times \sqrt{14}} = \frac{5}{7}$$

77. *(d)* Given point (2, 2, 2) and equation of plane $2x - y + 3z = 5$

∴ Required distance

$$= \left|\frac{ax_1 + by_1 + cz_1 - d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

$$= \left|\frac{2 \times 2 - 2 \times 1 + 3 \times 2 - 5}{\sqrt{(2)^2 + (-1)^2 + (3)^2}}\right|$$

$$= \left|\frac{3}{\sqrt{14}}\right|$$

$$= \frac{3\sqrt{14}}{14}$$

78. *(d)* Given equation of planes are $x = \sqrt{3}$ and $z = \sqrt{2}$

Now, the angle between two planes is given

$$\theta = \cos^{-1}\left|\frac{1 \times 0 + 0 \times 0 + 0 \times 1}{\sqrt{(1)^2 + (0)^2 + (0)^2}\sqrt{(0)^2 + (0)^2 + (1)^2}}\right|$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

79. *(a)* Three fair dice are rolled simultaneously.

Let, a, b, c are the number on the top of the dice. The size of sample space when three dice are rolled together is $6 \times 6 \times 6 = 216$

Since, number of ways in which 3 dice show same number is {(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)} = 6 ways

But hence only take min $\{a, b, c\} = 6$ i.e. one way.

So, the probability that min $(a, b, c) = 6$ is $\frac{1}{216}$.

80. *(c)* Given,

$P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$

We know that,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Now, $P(A/A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)}$

$$= \frac{P(A)}{P(A) + P(B) - P(A \cap B)}$$

$$= \frac{0.5}{0.5 + 0.4 - 0.2} = \frac{0.5}{0.7} = \frac{5}{7}$$

81. *(a)* Given, men in party = 37
and number of women in party = 33
Total number of persons in party = 37 + 33 = 70
Probability that prize goes to a women
$= \frac{\text{Total women in party}}{\text{Total persons}} = \frac{33}{70}$

82. *(d)* A fair coin tossed twice. Given that the first toss resulted is head. Then second toss possibility is {*H*, *T*}.
Hence, possible events {*HH*, *HT*}
Thus, probability that second toss also would result in head is 1/2.

83. *(d)* We know that, $C.V = \frac{\sigma}{\bar{x}} \times 100$

$C \cdot V = 75, \bar{x} = 44$

So, $75 = \frac{\sigma}{44} \times 100$

$\Rightarrow \quad \sigma = \frac{75 \times 44}{100} = \frac{3 \times 44}{4}$

$\sigma = 3 \times 11$

$\therefore \quad \sigma = 33$

84. *(b)* Total number of marbles contained in the urn = 4 + 3 + 3 = 10

Using formula, $^nC_r = \frac{n!}{r! \cdot n-r!}$

Number of ways selecting 3 marbles = $^{10}C_3$

$n(S) = \frac{10!}{7!3!}$

$= \frac{10 \times 9 \times 8 \times 7!}{7! \times 3 \times 2} = 120$

Now, probability of number of yellow marbles will be less than $2 = \frac{n(E)}{n(s)}$

Number of ways selecting of one yellow marble $= {}^3C_1 \times {}^7C_2 = 63$

Number of ways selecting of zero yellow marble $= {}^7C_3 = 35$

Thus, required probability $= \frac{63 + 35}{120}$

$= \frac{98}{120} = \frac{49}{60}$

85. *(d)* Given that, in a box there are four marbles and each of them is marked with distinct number from the set {1, 2, 5, 10}.
Thus, the sample outcomes is 18.

The probability of the sum of numbers equals to $18 = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$= \frac{3}{4 \times 8} = \frac{3}{32}$

86. *(b)* $\lim_{t \to 0} \left(\frac{(2t-3)(t-2)}{t} - \frac{3(t+2)}{t} \right)$

$\Rightarrow \quad \lim_{t \to 0} \frac{2t^2 - 4t - 3t + 6 - 3t - 6}{t}$

$\Rightarrow \quad \lim_{t \to 0} \frac{2t^2 - 10t}{t}$

$\Rightarrow \quad \lim_{t \to 0} t(2t - 10)$

$= 0 - 10 = -10$

87. *(a)* $\lim_{x \to -3^+} \left[x \cos \left(\frac{\pi}{3} x \right) \right]$

On putting $x = -3 + h$, we get

$\lim_{h \to 0} (-3 + h) \cos \left[\frac{\pi}{3} (-3 + h) \right]$

$\Rightarrow \quad \lim_{h \to 0} (-3 + h) \cos \left[-\pi + \frac{\pi h}{3} \right]$

$\Rightarrow \quad \lim_{h \to 0} -(-3 + h) \cos \frac{\pi h}{3}$

$\Rightarrow \quad -[(-3) + 0] \cos 0°$

$= 3 \times 1 = 3$

88. *(e)* We have,

$\lim_{x \to 0} \frac{\log(1 + x) + 1 - e^x}{4x^2 - 9x}$ $\left[\frac{0}{0} \text{ form} \right]$

$= \lim_{x \to 0} \frac{\frac{1}{1+x} - e^x}{8x - 9}$

$= \frac{\frac{1}{1} - e^0}{0 - 9} = 0$

89. *(d)* Given, $\lim_{t \to 0} \frac{\sin(t^2)}{t \sin(5t)}$

$\lim_{t \to 0} \left(\frac{\sin t^2}{t^2} \right) \frac{5t}{(\sin 5t) \times 5}$

$\Rightarrow \quad \lim_{t \to 0} \left(\frac{\sin t^2}{t^2} \right) \times \frac{1}{\left(\frac{\sin 5t}{5t} \right)} \times \frac{1}{5}$

$= \frac{1}{5}$

90. (c) Given, $f(x) = \begin{cases} 3x + 6, & x \geq c \\ x^2 - 3x - 1, & x < c \end{cases}$

According to the question,
f is continuous on R.
Then,

$$3c + 6 = c^2 - 3c - 1$$
$$\Rightarrow \quad c^2 - 6c - 7 = 0$$
$$\Rightarrow \quad (c - 7)(c + 1) = 0$$
$$\Rightarrow \quad c = -1, 7$$

91. (d) Given,

$$\lim_{x \to -2} \frac{3x^2 + ax - 2}{x^2 - x - 6}$$

$$= \frac{3(2)^2 - a \times 2 - 2}{(2)^2 - 2 - 6}$$
$$= \frac{10 - 2a}{0}$$

According to the question $\lim_{x \to -2} \frac{3x^2 + ax - 2}{x^2 - x - 6}$ is a finite number, then

$$10 - 2a = 0$$
$$\therefore \quad a = 5$$

92. (e) Given, $x = \sqrt{10^{\cos^{-1}\theta}}$ and $y = \sqrt{10^{\sin^{-1}\theta}}$

$x^2 = 10^{\cos^{-1}\theta}$ and $y^2 = 10^{\sin^{-1}\theta}$

Taking $\log_{10}$ on both sides,
$2\log_{10} x = \cos^{-1}\theta$ and $2\log_{10} y = \sin^{-1}\theta$

Now,

$$\frac{2}{x}\frac{dx}{d\theta} = \frac{-1}{\sqrt{1-\theta^2}} \quad \text{...(i)}$$
$$\frac{2}{y}\frac{dy}{d\theta} = \frac{1}{\sqrt{1-\theta^2}} \quad \text{...(ii)}$$

Divide Eq. (ii) by (i)Eq. (i), we get

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\theta^2}} \times \frac{\sqrt{1-\theta^2}}{-1} \times \frac{y}{x}$$
$$\frac{dy}{dx} = \frac{-y}{x}$$

93. (b) Given,

$$y = e^{3\log(2x+1)} \quad \text{...(i)}$$

Taking $\log_e$ on both sides, we get

$$\log_e y = 3\log(2x + 1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{(2x+1)} \times 2$$
$$\therefore \quad \frac{dy}{dx} = \frac{6}{(2x+1)}[e^{3\log(2x+1)}] \text{ [From Eq. (i)]}$$

94. (c) Given,
$x \sin y + y \sin x = \pi$

$$\sin y + x\cos y\frac{dy}{dx} + y\cos x + \frac{dy}{dx}\sin x = 0$$
$$\Rightarrow \quad \frac{dy}{dx}(x\cos y + \sin x) = -y\cos x \sin y$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{-(y\cos x + \sin y)}{(x\cos y + \sin x)}$$
$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{\left(\frac{\pi}{2}, \frac{\pi}{2}\right)} = \frac{-1}{1} = -1$$

95. (a) For differentiability at $x = \frac{\pi}{4}$

LHD $\lim_{x \to \frac{\pi}{4}} (\tan x) = \tan\frac{\pi}{4} = 1$

and RHD $\lim_{x \to \frac{\pi}{4}} (ax + b) = a\frac{\pi}{4} + b$

So, LHD = RHD

$$\Rightarrow \quad 1 = \frac{\pi}{4}a + b$$
$$\Rightarrow \quad b = 1 - \frac{\pi}{4}a$$

If $a = 2$, $b = 1 - \frac{\pi}{4} \times 2$

$$b = \frac{2 - \pi}{2}$$

96. (e) Given,

$$\frac{d}{dx}\left(\frac{1}{x}\frac{d^2}{dx^2}\left(\frac{1}{x^3}\right)\right)$$
$$= \frac{d}{dx}\left(\frac{1}{x}\frac{d}{dx}\left(\frac{d}{dx}\left(\frac{1}{x^3}\right)\right)\right)$$
$$= \frac{d}{dx}\left[\frac{1}{x}\frac{d}{dx}\left(\frac{-3}{x^4}\right)\right]$$
$$= \frac{d}{dx}\left[\frac{1}{x} \times \frac{(-3)(-4)}{x^5}\right]$$
$$= \frac{d}{dx}\left[\frac{12}{x^6}\right]$$
$$= \frac{-12 \times 6}{x^7} = -72x^{-7}$$

97. ***(b)*** Here, we have to calculate rate of change of volume with respect to diameter.

So, let r be the radius and V be the volume of sphere.

Then, $V = \frac{4}{3}\pi r^3$

Also, rate of change of diameter with respect to time is 3 cm/min.

$$\Rightarrow \quad \frac{d(2r)}{dt} = 3\text{cm/min}$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{3}{2}\text{cm/min}$$

Now, $\frac{dV}{dt} = \frac{4}{3} \times 3\pi r^2 \cdot \frac{dr}{dt}$

$$\frac{dV}{dt} = \frac{4}{3} \times 3 \times \pi \times (5)^2 \times \frac{3}{2} \quad [\because d = 10]$$

$$\frac{dV}{dt} = 150\,\pi\,\text{cm}^3\text{/min}$$

98. ***(a)*** Given, equation of circle,

$$(x-5)^2 + y^2 = 25$$

$$\Rightarrow \quad 2(x-5) + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-2(x-5)}{2y} \Rightarrow \frac{dy}{dx} = \frac{5-x}{y}$$

Equation of tangent at (2, 4)

$$\Rightarrow \quad y - 4 = \left(\frac{dy}{dx}\right)_{(2,4)}(x-2)$$

$$\Rightarrow \quad y - 4 = \frac{5-2}{4}(x-2)$$

$$\Rightarrow \quad 4y - 16 = 3x - 6$$

$$\therefore \quad 3x - 4y + 10 = 0$$

Hence the equation of tangent to the circle $3x - 4y + 10 = 0$

9. ***(c)*** Let $f(x) = 5\sin x \cdot \sin y$

$$f(x) = 5\sin x \sin(\pi - x) \quad [\because x + y = \pi]$$

$$f(x) = 5\sin x \cdot \sin x$$

$$f(x) = 5\sin^2 x$$

$$f'(x) = 10\sin x \cdot \cos x$$

$$f'(x) = 5\sin 2x$$

Now, $f'(x) = 0$, then we get value

$$\sin 2x = 0$$

$$\therefore \quad x = \frac{\pi}{2}$$

So, $f\left(\frac{\pi}{2}\right) = 5 \cdot \sin\frac{\pi}{2} \cdot \sin\frac{\pi}{2}$

Hence, maximum value of $5\sin x \sin y$ is 5.

100. ***(b)*** Given curve $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{\frac{1}{2\sqrt{x}}} = -2\sqrt{x}$$

$\therefore$ Slope of normal $= -2\sqrt{x}$

Slope of normal at point (25, 5) $= -2 \times 5 = -10$

Equation of normal at point (25, 5)

$$y - 5 = -10(x - 25)$$

$$y - 5 = -10x + 250$$

Since, normal to curve $y = \sqrt{x}$ at the point (25, 5) meets Y-axis i.e. $x = 0$

$$y - 5 = 250$$

$$y = 255$$

So, the required point is (0, 255)

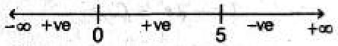
101. ***(d)*** We have, $f(x) = x^5 e^{-x}$

$$\therefore \quad f'(x) = -x^5 e^{-x} + 5x^4 e^{-x}$$

$$= e^{-x}[5x^4 - x^5]$$

So, $f'(x) > 0$ for increasing.

On drawing number lines as below



We can see that $f'(x)$ is increasing in $(-\infty, 5)$.

102. ***(e)*** Here, $y = 5x^2 + \alpha x + \beta$

$$y' = 10x + \alpha$$

So, $\quad m = 10 + \alpha$ $\quad$ ... (i)

Now, $x + 13y = 40$

Slope of $x + 13y = 40$ is $\frac{-1}{13}$ $\quad$ ... (ii)

From Eqs. (i) and (ii), we get

$$10 + \alpha = 13$$

$$\Rightarrow \quad \alpha = 3$$

At point (1, 3)

$$3 = 5(1)^2 + 3 \times 1 + \beta$$

$$3 = 5 + 3 + \beta$$

$$\beta = -5$$

Since, $\alpha \cdot \beta = -5 \times 3 = -15$

103. ***(a)*** Given, $f(x) = \cos x,\ x \in \left[0, \frac{\pi}{3}\right]$

Which is a trigonometric function.

So, $f(x)$ is continuous on $\left[0, \frac{\pi}{3}\right]$.

Also $f'(x) = -\sin x$, which exist for all $x \in \left[0, \frac{\pi}{3}\right]$

So, $f(x)$ is differentiable on $\left[0, \frac{\pi}{3}\right]$.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad -\sin c = \frac{\frac{1}{2} - 1}{\frac{\pi}{3}} \Rightarrow -\sin c = \frac{\frac{-1}{2}}{\frac{\pi}{3}}$$

$$\Rightarrow \quad \sin c = \frac{3}{2\pi}$$

$$\therefore \quad c = \sin^{-1}\left(\frac{3}{2\pi}\right)$$

104. ***(d)*** Let $I = \int \frac{e^{\frac{1}{\sqrt{t}}}}{t\sqrt{t}}\, dt$

Put $\quad \frac{1}{\sqrt{t}} = k \quad \ldots \text{(i)}$

$$-\frac{1}{2}\frac{1}{t\sqrt{t}}\, dt = dk$$

$$I = \int -2e^k dk$$

$$I = -2e^k + C$$

$$I = -2e^{\frac{1}{\sqrt{t}}} + C \quad \left[\text{from Eq. (i) } k = \frac{1}{\sqrt{t}}\right]$$

105. ***(d)*** Let $I = \int \frac{\sin^{25} x}{\cos^{27} x}\, dx$

$$I = \int \tan^{25} x \cdot \sec^2 x\, dx$$

Put $\tan x = k$

$$\sec^2 x\, dx = dk$$

$$I = \int k^{25} dk$$

$$I = \frac{k^{26}}{26} + C$$

$$I = \frac{\tan^{26} x}{26} + C \quad [\because k = \tan x]$$

106. ***(c)*** Clearly, the feasible region is the shaded region. Whose corner points are O, A, B and C. The co-ordinates of O, A, B and C are (0, 0), (20, 0), (10, 50) and (0, 60).

Now, let us find the value of z at corner points, as shown in the following table.

Corner points	Value of $z = 50x + 15y$
O (0, 0)	0
A (20, 0)	1000
B (10, 50)	1250 ← maximum
C (0, 60)	900

The maximum value of z is 1250 at B(10, 50).

107. ***(b)*** Let $I = \int \frac{1}{x^3}\sqrt{1 - \frac{1}{x^2}}\, dx$

Put $\quad 1 - \frac{1}{x^2} = t$

$$\frac{2}{x^3}\, dx = dt$$

Now, $I = \int \sqrt{t}\, \frac{dt}{2}$

$$I = \frac{2}{3} \times \frac{t^{3/2}}{2} + C$$

$$I = \frac{1}{3}\left(1 - \frac{1}{x^2}\right)^{3/2} + C$$

108. ***(e)*** Let $I = \int (\tan^2 2x - \cot^2 2x)\, dx$

$$I = \int (\tan 2x + \cot 2x)(\tan 2x - \cot 2x)\, dx$$

$$I = \int \left(\frac{1}{\cos 2x \cdot \sin 2x}\right)\left(\frac{\sin^2 2x - \cos^2 2x}{\cos 2x \cdot \sin 2x}\right) dx$$

$$I = \int 4 \times \frac{-\cos 4x}{(\sin^2 4x)}\, dx$$

$$I = -4\int \frac{\cos 4x}{\sin^2 4x}\, dx$$

Put $\sin 4x = k$

$4 \cos 4x\, dx = dk$

$$I = -4\int \frac{1}{k^2}\frac{dk}{4}$$

$$I = -\frac{k^{-2+1}}{-2+1} + C$$

$$I = \frac{1}{k} + C$$

$$I = \frac{1}{\sin 4x} + C$$

$$I = \frac{1}{2\sin 2x \cdot \cos 2x} + C$$

$$I = \frac{1}{2}\left[\frac{\sin^2 2x + \cos^2 2x}{\sin 2x \cdot \cos 2x}\right] + C$$

$$I = \frac{1}{2}[\tan 2x + \cot 2x] + C$$

109. *(a)* Let $I = \int \sin^3 x\,dx + \int \cos^2 x \sin x\,dx$

$$I = \int (\sin^3 x + \cos^2 x \sin x)\,dx$$

$$I = \int \sin x\,[\sin^2 x + \cos^2 x]\,dx$$

$$I = \int \sin x\,dx \Rightarrow I = -\cos x + C$$

110. *(d)* Let $I = \int \frac{dx}{x^2 - x}$

$$I = \int \frac{1}{x(x-1)}\,dx \Rightarrow I = \int \frac{1 + x - x}{x(x-1)}\,dx$$

$$I = \int \frac{1-x}{x(x-1)}\,dx + \int \frac{x}{x(x-1)}\,dx$$

$$I = \int \frac{-1}{x}\,dx + \int \frac{1}{x-1}\,dx$$

$$I = -\log|x| + \log|x-1| + C$$

$$I = \log\frac{|x-1|}{|x|} + C$$

111. *(e)* Let $I = \int_{\pi/6}^{\pi/2} \frac{\cot x}{\sin x}\,dx$

$$I = \int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin^2 x}\,dx$$

Put $\sin x = k \Rightarrow \cos x\,dx = dk$

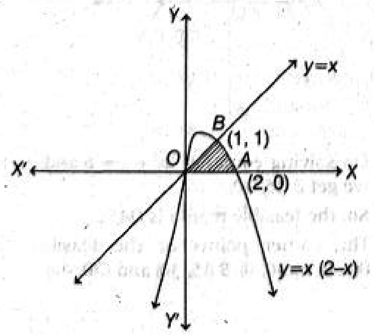
Lower limit when $x = \frac{\pi}{6}$, then $k = \frac{1}{2}$

Upper limit when $x = \frac{\pi}{2}$, then $k = 1$

$$I = \int_{1/2}^{1} \frac{1}{k^2}\,dx = \left[\frac{k^{-2+1}}{-2+1}\right]_{1/2}^{1}$$

$$= \left[\frac{-1}{k}\right]_{1/2}^{1} = -\left[\frac{1}{1} - \frac{1}{1} \times 2\right] = -[-1] = 1$$

112. *(d)* Given, the curve $y = x(2-x)$ and the line $y = x$



$$A = \int_0^1 x(2-x)\,dx + \int_1^2 x(2-x)\,dx - \int_0^1 x\,dx$$

$$A = \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^1 + \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_1^2 - \left[\frac{x^2}{2}\right]_0^1$$

$$A = \left[1 - \frac{1}{3}\right] + \left[4 - \frac{8}{3} - 1 + \frac{1}{3}\right] - \left[\frac{1}{2}\right]$$

$$A = \frac{4}{3} - \frac{1}{2} = \frac{8-3}{6} = \frac{5}{6}$$

113. *(d)* Given, $\int_{-1}^{2} [x - 2|x|]\,dx$

If $|x| = -x, x < 0 = x, x > 0$

So, $\int_{-1}^{0} [x - 2(-x)]dx + \int_0^2 (x - 2x)\,dx$

$$= \left[\frac{x^2}{2} + \frac{2x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{2x^2}{2}\right]_0^2$$

$$= -\left[\frac{1}{2} + 1\right] + [2 - 4] = \frac{-3}{2} - 2 = \frac{-7}{2}$$

114. *(e)* Let $I = \int_{-10}^{10} \frac{x^{10} \sin x}{\sqrt{1 + x^{10}}}\,dx$

Here, $f(x) = \frac{x^{10} \sin x}{\sqrt{1 + x^{10}}}$

Now, $f(-x) = -\frac{x^{10} \sin x}{\sqrt{1 + x^{10}}}$

$$f(-x) = -f(x)$$

So, $f(x)$ is an odd function.

$$I = \int_{-10}^{10} \frac{x^{10} \sin x}{\sqrt{1 + x^{10}}} = 0$$

115. *(c)* $f(x) = \begin{cases} \cos x & \text{for } x \ge 0 \\ 2x & \text{for } x < 0 \end{cases}$

Here, $\int_{-2}^{\pi/2} f(x)\,dx = \int_{-2}^{0} 2x + \int_0^{\pi/2} \cos x\,dx$

$$= \left[\frac{2x^2}{2}\right]_{-2}^{0} + [\sin x]_0^{\pi/2}$$

$$= -4 + 1 = -3$$

116. *(a)* Let $I = \int_0^{\pi/16} \cos 6x \cos 2x\,dx$

$$\cos 6x \cdot \cos 2x = \frac{1}{2}[\cos(6x + 2x) + \cos(6x - 2x)]$$

$$I = \frac{1}{2}\int_0^{\pi/16} \cos 8x + \frac{1}{2}\int_0^{\pi/16} \cos 4x\, dx$$

$$I = \frac{1}{2}\left[\frac{\sin 8x}{8} + \frac{\sin 4x}{4}\right]_0^{\pi/16}$$

$$I = \frac{1}{16} \times \sin\frac{\pi}{2} + \frac{1}{8}\sin\frac{\pi}{4}$$

$$I = \frac{1}{16} + \frac{1}{8} \times \frac{1}{\sqrt{2}}$$

$$I = \frac{1+\sqrt{2}}{16}$$

117. (b) Given differential equation, $\frac{dy}{dx} = xy^2$

$\Rightarrow \quad \frac{dy}{y^2} = x\, dx$

On integrating both sides, we get

$$\int \frac{dy}{y^2} = \int x\, dx$$

$\Rightarrow \quad \frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + C$

$\Rightarrow \quad \frac{-1}{y} = \frac{x^2}{2} + C$

$\Rightarrow \quad \frac{-1}{1} = 0 + C \qquad [\because y(0) = 1]$

$C = -1$

$\Rightarrow \quad \frac{-1}{y} = \frac{x^2}{2} - 1 \Rightarrow \frac{-1}{y} = \frac{x^2-2}{2}$

$\Rightarrow \quad y = \frac{2}{2-x^2}$

118. (d) Given, differential equation

$\Rightarrow \quad (x^2y^2 + y)\, dx - (x - 2x^3y)dy = 0$

$\Rightarrow \quad x^2y^2\, dx + y\, dx - x\, dy + 2x^3y dy = 0$

$\Rightarrow \quad dx\,(x^2y^2) + 2x^3y\, dy = xdy - ydx$

$\Rightarrow \quad y^2dx + 2xy\, dy = \frac{x\, dy - y\, dx}{x^2}$

$\Rightarrow \quad y^2x = \frac{y}{x} + C$

$\Rightarrow \quad xy^2 - \frac{y}{x} = C$

119. (c) Given, $4x\, dy - e^{-2y}dy + dx = 0$

$$4x - e^{-2y} + \frac{dx}{dy} = 0$$

$$\frac{dx}{dy} + 4x = e^{-2y}$$

This is of the form $\frac{dx}{dy} + Px = Q$

Here, $P = 4$ and $Q = e^{-2y}$

$$\text{IF} = e^{\int P\, dy}$$

$$\text{IF} = e^{\int 4\, dy}$$

$$\text{IF} = e^{4y}$$

120. (a) Subject to constraints

$$2x + 3y \le 120$$

$$2x + y \le 60$$

$$x, y \ge 0$$

Firstly, draw the graph of the line

$2x + 3y = 120$

x	0	60
y	40	0

Put (0, 0) in the inequality $2x + 3y \le 120$

we get, $0 \le 120$, which is true.

So, the half plane is toward the origin.

Secondly draw the graph of the line

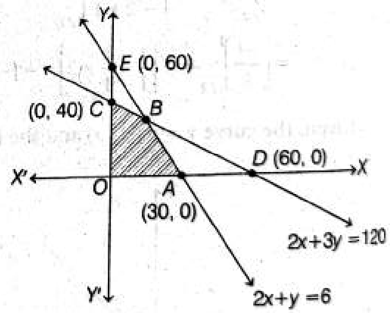
$2x + y = 60$

x	0	30
y	60	0

Put (0, 0) in the inequality $2x + y \le 60$

we get, $0 \le 60$, which is true.

So, the half plane is toward the origin.



On solving equation $2x + y = 6$ and $2x + 3y = 120$

we get $B\,(15, 30)$.

So, the feasible region is $OABC$.

The corner points of the feasible region are $O(0, 0)$, $A(30, 0)$, $B\,(15, 30)$ and $C(0, 40)$.